

# ALGEBRA

## Factors and Zeros of Polynomials

Let  $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$  be a polynomial. If  $p(a) = 0$ , then  $a$  is a zero of the polynomial and a solution of the equation  $p(x) = 0$ . Furthermore,  $(x - a)$  is a factor of the polynomial.

## Fundamental Theorem of Algebra

An  $n$ th degree polynomial has  $n$  (not necessarily distinct) zeros. Although all of these zeros may be imaginary, a real polynomial of odd degree must have at least one real zero.

## Quadratic Formula

If  $p(x) = ax^2 + bx + c$ , and  $0 \leq b^2 - 4ac$ , then the real zeros of  $p$  are  $x = (-b \pm \sqrt{b^2 - 4ac})/2a$ .

## Special Factors

$$x^2 - a^2 = (x - a)(x + a)$$

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$

$$x^4 - a^4 = (x^2 - a^2)(x^2 + a^2)$$

## Binomial Theorem

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \dots + nxy^{n-1} + y^n$$

$$(x - y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \dots \pm nxy^{n-1} \mp y^n$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$$

## Rational Zero Theorem

If  $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$  has integer coefficients, then every rational zero of  $p$  is of the form  $x = r/s$ , where  $r$  is a factor of  $a_0$  and  $s$  is a factor of  $a_n$ .

## Factoring by Grouping

$$acx^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cs + d) = (ax^2 + b)(cx + d)$$

## Arithmetic Operations

$$ab + ac = a(b + c)$$

$$\left(\frac{a}{b}\right) \left(\frac{c}{d}\right) = \left(\frac{a}{b}\right) \left(\frac{d}{c}\right) = \frac{ad}{bc}$$

$$a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\left(\frac{a}{b}\right) \left(\frac{b}{c}\right) = \frac{a}{c}$$

$$\frac{a-b}{c-d} = \frac{b-a}{d-c}$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{ab + ac}{a} = b + c$$

## Exponents and Radicals

$$a^0 = 1, \quad a \neq 0$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$(ab)^x = a^x b^x$$

$$\sqrt[n]{a^m} = a^{m/n}$$

$$a^x a^y = a^{x+y}$$

$$a^{-x} = \frac{1}{a^x}$$

$$\sqrt{a} = a^{1/2}$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

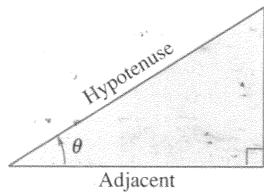
$$\sqrt[n]{a} = a^{1/n}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

# TRIGONOMETRY

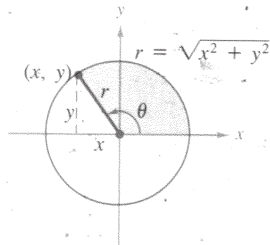
## Definition of the Six Trigonometric Functions

Right triangle definitions, where  $0 < \theta < \pi/2$ .



$$\begin{aligned} \sin \theta &= \frac{\text{opp.}}{\text{hyp.}} & \csc \theta &= \frac{\text{hyp.}}{\text{opp.}} \\ \cos \theta &= \frac{\text{adj.}}{\text{hyp.}} & \sec \theta &= \frac{\text{hyp.}}{\text{adj.}} \\ \tan \theta &= \frac{\text{opp.}}{\text{adj.}} & \cot \theta &= \frac{\text{adj.}}{\text{opp.}} \end{aligned}$$

Circular function definitions, where  $\theta$  is any angle.



$$\begin{aligned} \sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$

## Reciprocal Identities

$$\begin{aligned} \sin x &= \frac{1}{\csc x} & \sec x &= \frac{1}{\cos x} & \tan x &= \frac{1}{\cot x} \\ \csc x &= \frac{1}{\sin x} & \cos x &= \frac{1}{\sec x} & \cot x &= \frac{1}{\tan x} \end{aligned}$$

## Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

## Pythagorean Identities

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x & 1 + \cot^2 x &= \csc^2 x \end{aligned}$$

## Cofunction Identities

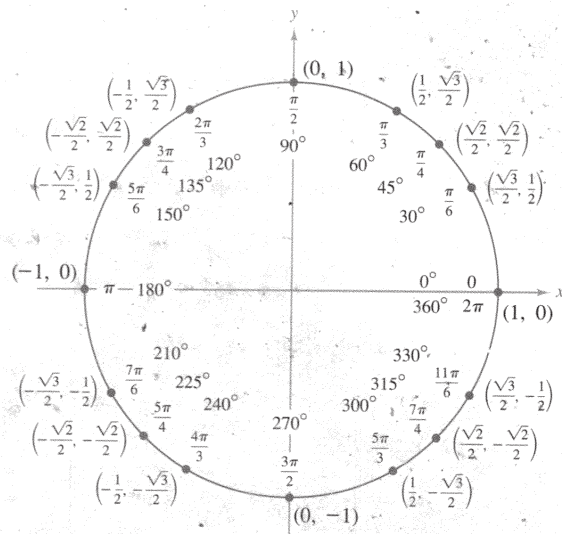
$$\begin{aligned} \sin\left(\frac{\pi}{2} - x\right) &= \cos x & \cos\left(\frac{\pi}{2} - x\right) &= \sin x \\ \csc\left(\frac{\pi}{2} - x\right) &= \sec x & \tan\left(\frac{\pi}{2} - x\right) &= \cot x \\ \sec\left(\frac{\pi}{2} - x\right) &= \csc x & \cot\left(\frac{\pi}{2} - x\right) &= \tan x \end{aligned}$$

## Reduction Formulas

$$\begin{aligned} \sin(-x) &= -\sin x & \cos(-x) &= \cos x \\ \csc(-x) &= -\csc x & \tan(-x) &= -\tan x \\ \sec(-x) &= \sec x & \cot(-x) &= -\cot x \end{aligned}$$

## Sum and Difference Formulas

$$\begin{aligned} \sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} \end{aligned}$$



## Double-Angle Formulas

$$\begin{aligned} \sin 2u &= 2 \sin u \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u} \end{aligned}$$

## Power-Reducing Formulas

$$\begin{aligned} \sin^2 u &= \frac{1 - \cos 2u}{2} \\ \cos^2 u &= \frac{1 + \cos 2u}{2} \\ \tan^2 u &= \frac{1 - \cos 2u}{1 + \cos 2u} \end{aligned}$$

## Sum-to-Product Formulas

$$\begin{aligned} \sin u + \sin v &= 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v &= 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v &= 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v &= -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \end{aligned}$$

## Product-to-Sum Formulas

$$\begin{aligned} \sin u \sin v &= \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2} [\sin(u+v) - \sin(u-v)] \end{aligned}$$