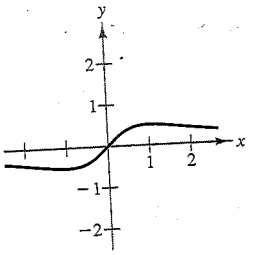
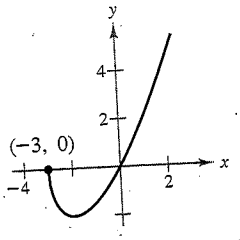
In Exercises 57–60, find the interval(s) for which the function is continuous.

57. $f(x) = \dfrac{x}{x^2 + 1}$

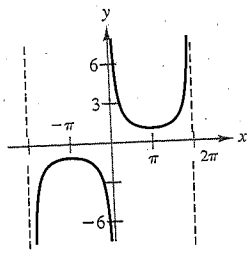
58. $f(x) = x\sqrt{x + 3}$

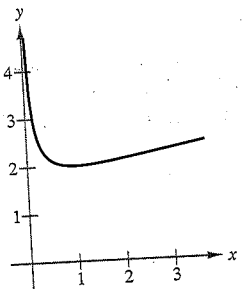




59. $f(x) = \csc\dfrac{x}{2}$

60. $f(x) = \dfrac{x + 1}{\sqrt{x}}$





C **In Exercises 61–64, use a graphing utility to graph the function. Then use the graph to determine any x-values at which the function is not continuous.**

61. $f(x) = [\![x]\!] - x$

62. $h(x) = \dfrac{1}{x^2 - x - 2}$

63. $g(x) = \begin{cases} 2x - 4, & x \le 3 \\ x^2 - 2x, & x > 3 \end{cases}$

64. $f(x) = \begin{cases} \dfrac{\cos x - 1}{x}, & x < 0 \\ 5x, & x \ge 0 \end{cases}$

C *Essay* **In Exercises 65 and 66, use a graphing utility to graph the function on the interval $[-4, 4]$. Does the graph of the function appear continuous on this interval? *Is* the function continuous on $[-4, 4]$? Write a short paragraph about the importance of examining a function analytically as well as graphically.**

65. $f(x) = \dfrac{\sin x}{x}$

66. $f(x) = \dfrac{x^3 - 8}{x - 2}$

Essay **In Exercises 67 and 68, give a written explanation of why the function has a zero in the indicated interval.**

67. $f(x) = x^2 - 4x + 3, \quad [2, 4]$

68. $f(x) = x^3 + 3x - 2, \quad [0, 1]$

C **In Exercises 69 and 70, use the Intermediate Value Theorem and a graphing utility to approximate the zero of the function in the interval $[0, 1]$. Repeatedly "zoom in" on the graph of the function to approximate the zero accurate to two decimal places.**

69. $f(x) = x^3 + x - 1$

70. $f(x) = x^3 + 3x - 2$

In Exercises 71–74, verify that the Intermediate Value Theorem applies in the indicated interval and find the value of c guaranteed by the theorem.

71. $f(x) = x^2 + x - 1, \quad [0, 5], \quad f(c) = 11$

72. $f(x) = x^2 - 6x + 8, \quad [0, 3], \quad f(c) = 0$

73. $f(x) = x^3 - x^2 + x - 2, \quad [0, 3], \quad f(c) = 4$

74. $f(x) = \dfrac{x^2 + x}{x - 1}, \quad \left[\dfrac{5}{2}, 4\right], \quad f(c) = 6$

75. *Salary Contract* A union contract guarantees a 9 percent increase for 5 years. For an initial annual salary of \$28,500, the salary S is

$$S = 28{,}500(1.09)^{[\![t]\!]}$$

where $t = 0$ corresponds to 1990. Sketch a graph of this function and discuss its continuity.

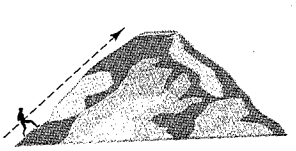
76. *Telephone Rates* A dial-direct long distance call between two cities costs \$1.04 for the first 2 minutes and \$0.36 for each additional minute or fraction thereof. Use the greatest integer function to write the cost C of a call in terms of the time t (in minutes). Sketch a graph of this function and discuss its continuity.

77. *Inventory Management* The number of units in inventory in a small company is

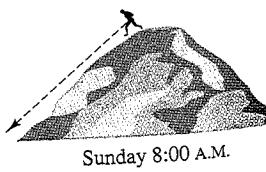
$$N(t) = 25\left(2\left[\!\left[\frac{t + 2}{2}\right]\!\right] - t\right), \qquad 0 \le t \le 12$$

where the real number t is the time in months. Sketch the graph of this function and discuss its continuity. How often must this company replenish its inventory?

78. *Déjà Vu* At 8:00 A.M. on Saturday a man begins running up the side of a mountain to his weekend campsite (see figure). On Sunday morning at 8:00 A.M. he runs back down the mountain. It takes him 20 minutes to run up, but only 10 minutes to run down. At some point on the way down, he realizes that he passed the same place at exactly the same time on Saturday. Prove that he is correct. [*Hint:* Let $s(t)$ and $r(t)$ be the position functions for the run up and down, respectively, and apply the Intermediate Value Theorem to the function $f(t) = s(t) - r(t)$.]



Saturday 8:00 A.M.



Sunday 8:00 A.M.

FIGURE FOR 78