EXERCISES for Section 5.2

TECHNOLOGY Laboratory Guide Lab 5.2

In Exercises 1–18, find the indefinite integral.

1. $\displaystyle\int \frac{1}{x+1}\,dx$
2. $\displaystyle\int \frac{1}{x-5}\,dx$
3. $\displaystyle\int \frac{1}{3-2x}\,dx$
4. $\displaystyle\int \frac{1}{6x+1}\,dx$
5. $\displaystyle\int \frac{x}{x^2+1}\,dx$
6. $\displaystyle\int \frac{x^2}{3-x^3}\,dx$
7. $\displaystyle\int \frac{x^2-4}{x}\,dx$
8. $\displaystyle\int \frac{x}{\sqrt{9-x^2}}\,dx$
9. $\displaystyle\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx$
10. $\displaystyle\int \frac{x+3}{x^2+6x+7}\,dx$
11. $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$
12. $\displaystyle\int \frac{1}{x\ln(x^2)}\,dx$
13. $\displaystyle\int \frac{1}{\sqrt{x+1}}\,dx$
14. $\displaystyle\int \frac{1}{x^{2/3}(1+x^{1/3})}\,dx$
15. $\displaystyle\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx$
16. $\displaystyle\int \frac{1}{1+\sqrt{2x}}\,dx$
17. $\displaystyle\int \frac{2x}{(x-1)^2}\,dx$
18. $\displaystyle\int \frac{x(x-2)}{(x-1)^3}\,dx$

In Exercises 19–26, find the indefinite integral of the trigonometric function.

19. $\displaystyle\int \frac{\cos\theta}{\sin\theta}\,d\theta$
20. $\displaystyle\int \tan 5\theta\,d\theta$
21. $\displaystyle\int \csc 2x\,dx$
22. $\displaystyle\int \sec\frac{x}{2}\,dx$
23. $\displaystyle\int \frac{\cos t}{1+\sin t}\,dt$
24. $\displaystyle\int \frac{\sin x}{1+\cos x}\,dx$
25. $\displaystyle\int \frac{\sec x\tan x}{\sec x-1}\,dx$
26. $\displaystyle\int (\sec t+\tan t)\,dt$

In Exercises 27–30, solve the differential equation.

27. $\displaystyle\frac{dy}{dx}=\frac{3}{2-x}$
28. $\displaystyle\frac{dy}{dx}=\frac{2x}{x^2-9}$
29. $\displaystyle\frac{ds}{d\theta}=\tan 2\theta$
30. $\displaystyle\frac{dr}{dt}=\frac{\sec^2 t}{\tan t+1}$

In Exercises 31–38, evaluate the definite integral.

31. $\displaystyle\int_0^4 \frac{5}{3x+1}\,dx$
32. $\displaystyle\int_{-1}^1 \frac{1}{5-2x}\,dx$
33. $\displaystyle\int_1^e \frac{(1+\ln x)^2}{x}\,dx$
34. $\displaystyle\int_e^{e^2} \frac{1}{x\ln x}\,dx$
35. $\displaystyle\int_0^2 \frac{x^2-2}{x+1}\,dx$
36. $\displaystyle\int_0^1 \frac{x-1}{x+1}\,dx$
37. $\displaystyle\int_1^2 \frac{1-\cos\theta}{\theta-\sin\theta}\,d\theta$
38. $\displaystyle\int_{0.1}^{0.2} (\csc 2\theta-\cot 2\theta)^2\,d\theta$

C **In Exercises 39–44, use a symbolic integration utility to evaluate the integral.**

39. $\displaystyle\int \frac{1}{1+\sqrt{x}}\,dx$
40. $\displaystyle\int \frac{1-\sqrt{x}}{1+\sqrt{x}}\,dx$
41. $\displaystyle\int \cos(1-x)\,dx$
42. $\displaystyle\int \frac{\tan^2 2x}{\sec 2x}\,dx$
43. $\displaystyle\int_{\pi/4}^{\pi/2} (\csc x-\sin x)\,dx$
44. $\displaystyle\int_{-\pi/4}^{\pi/4} \frac{\sin^2 x-\cos^2 x}{\cos x}\,dx$

In Exercises 45–48, show that the two formulas are equivalent.

45. $\displaystyle\int \tan x\,dx = -\ln|\cos x| + C$

$\displaystyle\int \tan x\,dx = \ln|\sec x| + C$

46. $\displaystyle\int \cot x\,dx = \ln|\sin x| + C$

$\displaystyle\int \cot x\,dx = -\ln|\csc x| + C$

47. $\displaystyle\int \sec x\,dx = \ln|\sec x+\tan x| + C$

$\displaystyle\int \sec x\,dx = -\ln|\sec x-\tan x| + C$

48. $\displaystyle\int \csc x\,dx = -\ln|\csc x+\cot x| + C$

$\displaystyle\int \csc x\,dx = \ln|\csc x-\cot x| + C$

In Exercises 49–52, find $F'(x)$.

49. $\displaystyle F(x)=\int_1^x \frac{1}{t}\,dt$
50. $\displaystyle F(x)=\int_0^x \tan t\,dt$
51. $\displaystyle F(x)=\int_x^{3x} \frac{1}{t}\,dt$
52. $\displaystyle F(x)=\int_1^{x^2} \frac{1}{t}\,dt$

Approximation **In Exercises 53 and 54, determine which best approximates the area of the region between the x-axis and the function over the given interval. (Make your selection on the basis of a sketch of the region and *not* by performing any calculations.)**

53. $f(x)=\sec x, \quad [0, 1]$

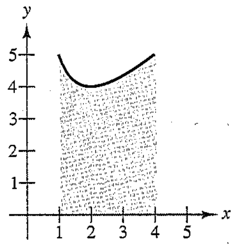
a. 6 **b.** -6 **c.** $\frac{1}{2}$ **d.** 1.25 **e.** 3

54. $\displaystyle f(x)=\frac{2x}{x^2+1}, \quad [0, 4]$

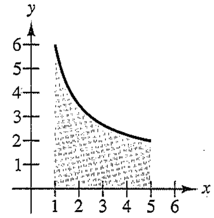
a. 3 **b.** 7 **c.** -2 **d.** 5 **e.** 1

Area **In Exercises 55 and 56, find the area of the indicated region.**

55. $y = \dfrac{x^2 + 4}{x}$



56. $y = \dfrac{x + 5}{x}$



57. *Population Growth* A population of bacteria is changing at the rate of

$$\frac{dP}{dt} = \frac{3000}{1 + 0.25t}$$

where t is the time in days. The initial population (when $t = 0$) is 1000. Write an equation that gives the population at any time t, and then find the population when $t = 3$ days.

58. *Heat Transfer* Find the time required for an object to cool from 300° to 250° by evaluating

$$t = \frac{10}{\ln 2}\int_{250}^{300} \frac{1}{T - 100}\,dT.$$

59. *Average Price* The demand equation for a product is given by

$$p = \frac{90{,}000}{400 + 3x}.$$

Find the *average* price p on the interval $40 \le x \le 50$.

60. *Sales* The rate of change in sales S is inversely proportional to time t $(t > 1)$ measured in weeks. Find S as a function of t if sales after 2 and 4 weeks were 200 and 300 units, respectively.

True or False **In Exercises 61–64, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

61. $(\ln x)^{1/2} = \frac{1}{2}(\ln x)$

62. $\displaystyle\int \ln x\,dx = \left(\frac{1}{x}\right) + C$

63. $\displaystyle\int \frac{1}{x}\,dx = \ln|cx|, \quad \text{for } c \neq 0$

64. $\displaystyle\int_{-1}^{e} \frac{1}{x} = \ln|x|\Big]_{-1}^{e} = \ln e - \ln 1 = 1$

65. Graph the function

$$f_k(x) = \frac{x^k - 1}{k}$$

for $k = 1, 0.5$, and 0.1 on $[0, 10]$. Find $\displaystyle\lim_{k \to 0} f_k(x)$.

CAREER INTERVIE

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At the CDC, I direct a group of scientists wh trained in mathematics, statistics, and epide ogy, and whose job it is to reduce the incider preventable diseases, disabilities, and injurie analyzing information about the numbers of ple infected by a particular disease, for instanc can assess the need for a specific vaccinatior gram. Furthermore, our data analysis on ir associated with automobile crashes has led t ommendations on laws governing required features for automobiles, such as seat belts o tent of gasoline.

Because much of our work at CDC is in stat my understanding of the calculus that is the of many statistical tests is an enormous as assure policymakers that the application method is valid and that incorrect conclusic inappropriate recommendations are avoidec careful and systematic approach to problem underlies calculus is essential to prevention a tervention efforts in public health, and to th ceptance of public health policy.

EXERCISES for Section 5.8

In Exercises 1–26, evaluate the integral.

1. $\int_0^{1/6} \frac{1}{\sqrt{1-9x^2}}\,dx$

2. $\int_0^1 \frac{dx}{\sqrt{4-x^2}}$

3. $\int_0^{\sqrt{3}/2} \frac{1}{1+4x^2}\,dx$

4. $\int_{\sqrt{3}}^{3} \frac{1}{9+x^2}\,dx$

5. $\int \frac{1}{x\sqrt{4x^2-1}}\,dx$

6. $\int \frac{1}{4+(x-1)^2}\,dx$

7. $\int \frac{x^3}{x^2+1}\,dx$

8. $\int \frac{x^4-1}{x^2+1}\,dx$

9. $\int \frac{1}{\sqrt{1-(x+1)^2}}\,dx$

10. $\int \frac{t}{t^4+16}\,dt$

11. $\int \frac{t}{\sqrt{1-t^4}}\,dt$

12. $\int \frac{1}{x\sqrt{x^4-4}}\,dx$

13. $\int \frac{\arctan x}{1+x^2}\,dx$

14. $\int \frac{1}{(x-1)\sqrt{(x-1)^2-4}}\,dx$

15. $\int_0^{1/\sqrt{2}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$

16. $\int_0^{1/\sqrt{2}} \frac{\arccos x}{\sqrt{1-x^2}}\,dx$

17. $\int_{-1/2}^{0} \frac{x}{\sqrt{1-x^2}}\,dx$

18. $\int_{-\sqrt{3}}^{0} \frac{x}{1+x^2}\,dx$

19. $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

20. $\int \frac{\cos x}{\sqrt{4-\sin^2 x}}\,dx$

21. $\int \frac{1}{9+(x-3)^2}\,dx$

22. $\int \frac{x+1}{x^2+1}\,dx$

23. $\int \frac{e^{2x}}{4+e^{4x}}\,dx$

24. $\int_1^2 \frac{1}{3+(x-2)^2}\,dx$

25. $\int_{\pi/2}^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$

26. $\int_0^{\pi} \frac{\cos x}{1+\sin^2 x}\,dx$

In Exercises 27–40, evaluate the integral. (Complete the square, if necessary.)

27. $\int_0^2 \frac{dx}{x^2-2x+2}$

28. $\int_{-3}^{-1} \frac{dx}{x^2+6x+13}$

29. $\int \frac{2x}{x^2+6x+13}\,dx$

30. $\int \frac{2x-5}{x^2+2x+2}\,dx$

31. $\int \frac{1}{\sqrt{-x^2-4x}}\,dx$

32. $\int \frac{1}{\sqrt{-x^2+2x}}\,dx$

33. $\int \frac{x+2}{\sqrt{-x^2-4x}}\,dx$

34. $\int \frac{x-1}{\sqrt{x^2-2x}}\,dx$

35. $\int_2^3 \frac{2x-3}{\sqrt{4x-x^2}}\,dx$

36. $\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx$

37. $\int \frac{x}{x^4+2x^2+2}\,dx$

38. $\int \frac{x}{\sqrt{9+8x^2}}$

39. $\int \frac{1}{\sqrt{-16x^2+16x-3}}\,dx$

40. $\int \frac{1}{(x-1)\sqrt{9x^2-18x+5}}\,dx$

In Exercises 41–44, use substitution to evaluat

41. $\int \frac{\sqrt{x-1}}{x}\,dx$

42. $\int \frac{\sqrt{x-2}}{x+1}\,dx$

43. $\int \sqrt{e^t-3}\,dt$

44. $\int \frac{1}{t\sqrt{t}+\sqrt{}}$

In Exercises 45–48, solve the differential equa

45. $(1+x^6)y' = 3x^2$

46. $\sqrt{1-9x^2}\,y'$

47. $\frac{dy}{dx} = \frac{xy}{1+x^2}$

48. $\frac{dy}{dx} = x\sec y$

In Exercises 49–52, find the area of the region b graphs of the given equations.

49. $y = \frac{1}{1+x^2}$, $y = 0$, $x = 0$, $x = 1$

50. $y = \frac{1}{\sqrt{4-x^2}}$, $y = 0$, $x = 0$, $x = 1$

51. $y = \frac{1}{x^2-2x+5}$, $y = 0$, $x = 1$, $x = 3$

52. $y = \frac{1}{\sqrt{3+2x-x^2}}$, $y = 0$, $x = 0$, $x = 2$

53. Consider the function

$$F(x) = \frac{1}{2}\int_x^{x+2} \frac{2}{t^2+1}\,dt.$$

a. Write a short paragraph giving a geometr of the function F relative to the function

$$f(x) = \frac{2}{x^2+1}.$$

Use what you have written to guess the va make F maximum.

b. Perform the specified integration to fin form of $F(x)$. Use calculus to locate the va make F maximum and compare the result in part a.

54. *Approximation* Determine which value be the area of the region between the x-axis a $f(x) = 1/\sqrt{1-x^2}$ over the given interv (Make your selection on the basis of a sketch *not* by performing any calculations.)

a. 4 **b.** -3 **c.** 1 **d.** 2

C **55.** *Vertical Motion* An object is projected upward from ground level with an initial velocity of 500 feet per second. In this exercise, the goal is to analyze the motion of the object during its upward flight.

a. If air resistance is neglected, express the velocity of the object as a function of time. Sketch the graph of this function.

b. Use the result of part a to find the position function and determine the maximum height attained by the object.

c. If the air resistance is proportional to the square of the velocity, you obtain the equation

$$\frac{dv}{dt} = -(32 + kv^2)$$

where 32 feet per second per second is the acceleration due to gravity and k is a constant. Find the velocity as a function of time by solving the equation

$$\int \frac{dv}{32 + kv^2} = -\int dt.$$

d. Use a graphing utility to graph the velocity function v from part c if $k = 0.001$. Use the graph to approximate the time t_0 at which the object reaches its maximum height.

e. Use Simpson's Rule with $n = 10$ to approximate the integral

$$\int_0^{t_0} v(t)\, dt$$

where $v(t)$ and t_0 are those found in part d. This is the approximation of the maximum height of the object.

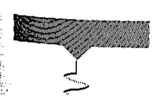
f. Explain the difference between the results of part b and part e.

FOR FURTHER INFORMATION For more information on this topic, see "What Goes Up Must Come Down; Will Air Resistance Make It Return Sooner, or Later?" by John Lekner in the January, 1982 issue of *Mathematics Magazine*.

56. *Harmonic Motion* A weight of mass m is attached to a spring and oscillates with simple harmonic motion (see figure). By Hooke's Law we can determine that

$$\int \frac{dy}{\sqrt{A^2 - y^2}} = \int \sqrt{\frac{k}{m}}\, dt$$

where A is the maximum displacement, t is the time, and k is a constant. Find y as a function of t, given that $y = 0$ when $t = 0$.



57. a. Show that

$$\int_0^1 \frac{4}{1 + x^2}\, dx = \pi.$$

b. Approximate the number π using Simpson's Rule (with $n = 6$) and the integral in part a.

58. Verify the following rules by differentiating ($a > 0$).

a. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

b. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

In Exercises 59–62, determine which of the given integrals can be evaluated using the basic integration rules you have studied so far.

59. a. $\displaystyle\int \frac{1}{\sqrt{1 - x^2}}\, dx$ **b.** $\displaystyle\int \frac{x}{\sqrt{1 - x^2}}\, dx$ **c.** $\displaystyle\int \frac{1}{x\sqrt{1 - x^2}}\, dx$

60. a. $\displaystyle\int e^{x^2}\, dx$ **b.** $\displaystyle\int xe^{x^2}\, dx$ **c.** $\displaystyle\int \frac{1}{x^2}e^{1/x}\, dx$

61. a. $\displaystyle\int \sqrt{x - 1}\, dx$ **b.** $\displaystyle\int x\sqrt{x - 1}\, dx$ **c.** $\displaystyle\int \frac{x}{\sqrt{x - 1}}\, dx$

62. a. $\displaystyle\int \frac{1}{1 + x^4}\, dx$ **b.** $\displaystyle\int \frac{x}{1 + x^4}\, dx$ **c.** $\displaystyle\int \frac{x^3}{1 + x^4}\, dx$

63. Sketch the region whose area is represented by the integral

$$\int_0^1 \arcsin(x)\, dx$$

and approximate its value.

64. Graph $y_1 = \dfrac{x}{1 + x^2}$, $y_2 = \arctan x$, and $y_3 = x$ on $[0, 10]$. Prove that

$$\frac{x}{1 + x^2} < \arctan x < x \quad \text{for} \quad x > 0.$$