3.1

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1] $$\lim_{h \to 0} \frac{\frac{x}{x}\frac{1}{x+h} - \frac{1}{x}\frac{x+h}{x+h}}{h}$$

$$\lim_{h \to 0} \frac{\frac{\cancel{x} - (\cancel{x}+h)}{x(x+h)}}{h}$$

$$\lim_{h \to 0} \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h \to 0} \frac{-1}{x(x+h)}$$

$$-\frac{1}{x \cdot x}$$

$$-\frac{1}{x^2}$$

$$@\ x = 2,\ f'(2) = \frac{-1}{(2)^2} = \frac{-1}{4}$$

5] $$\lim_{x \to 2} \frac{\frac{2}{2}\frac{1}{x} - \frac{1}{2}\frac{x}{x}}{x-2}$$

$$\lim_{x \to 2} \frac{\frac{2-x}{2x}}{x-2}$$

$$\lim_{x \to 2} \frac{\overset{-(\cancel{-2+x})}{2-x}}{2x} \cdot \frac{1}{\cancel{x-2}}$$

$$\lim_{x \to 2} \frac{-1}{2x} = \frac{-1}{4}$$

3.1

7] $\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2}$

$\lim_{x \to 3} \frac{x+1-4}{(x-3)[\sqrt{x+1} + 2]}$

$\lim_{x \to 3} \frac{\cancel{x-3}}{(\cancel{x-3})[\sqrt{x+1} + 2]}$

$\lim_{x \to 3} \frac{1}{\sqrt{x+1} + 2} = \frac{1}{4}$

17] $f(2) = 3 \quad f'(2) = 5$

$(2,3) \quad m = 5$ ✓ $\quad m = -\frac{1}{5}$ ✓

$y = mx + b$

$3 = 5(2) + b$

$3 = 10 + b$

$-10 \quad -10$

$-7 = b$ ✓

$y = 5x - 7$

$y = mx + b$

$3 = -\frac{1}{5}(2) + b$

$3 = \frac{-2}{5} + b$

$+\frac{2}{5} \quad +\frac{2}{5}$

$\frac{17}{5} = b$ ✓

$y = -\frac{1}{5}x + \frac{17}{5}$

3.1

FOIL

F O I L E D

19) $\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

$(x+h)(x+h)(x+h)$

$\lim_{h \to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}$

$\lim_{h \to 0} \frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$

$3x^2$

$f'(1) = 3(1)^2 = 3$

$m = -\frac{1}{3}$

$y - \cancel{1} = 3(x-1)$ $+1$ $+1$

$y = 3x - 3 + 1$

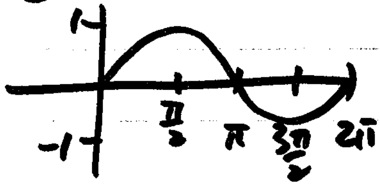
$\boxed{(a)\ y = 3x - 2}$

$y - \cancel{1} = -\frac{1}{3}(x-1)$ $+1$ $+1$

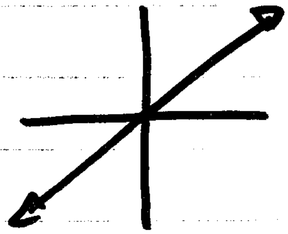
$y = -\frac{1}{3}x + \frac{1}{3} + 1$

$\boxed{(b)\ y = -\frac{1}{3}x + \frac{4}{3}}$

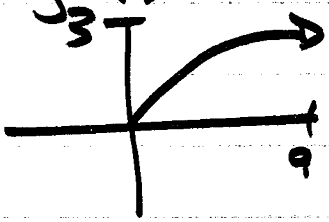
25) $y = \sin x$



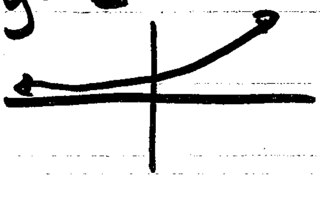
$y = x$



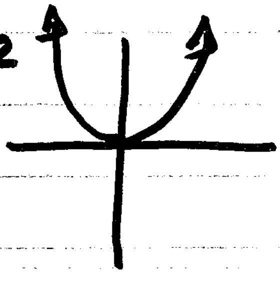
$y = \sqrt{x}$



$y = e^x$



$y = x^2$



3.1

31) $f(x) = \begin{cases} x^2+x, & x \le 1 \\ 3x-2, & x > 1 \end{cases}$ $\quad f'(x) = \begin{cases} 2x+1, & x \le 1 \\ 3, & x > 1 \end{cases}$

right-hand derivative at a (left endpoint)

$\lim_{h \to 0^+} \frac{f(a+h)-f(a)}{h}$

lef-hand derivative at b (right endpoint)

$\lim_{h \to 0^-} \frac{f(b+h)-f(b)}{h}$

[1, 1]

$\lim_{h \to 0^+} \frac{3(1+h)-2-2}{h}$ $\quad \frac{1}{0} \quad 0\overline{)1}$

$\lim_{h \to 0^+} \frac{3+3h-4}{h}$

$\lim_{h \to 0^+} \frac{3h-1}{h}$

$\lim_{h \to 0^+} 3 - \frac{1}{h} = -\infty$

3.1

32) $f(x) = \begin{cases} x^3, & x \leq 1 \\ 3x, & x > 1 \end{cases}$ $a=1$

$\lim_{h \to 0^+} \frac{3(1+h)-1}{h}$

$\lim_{h \to 0^+} \frac{3+3h-1}{h}$

$\lim_{h \to 0^+} \frac{2+3h}{h}$

$\lim_{h \to 0^+} \frac{2}{h}+3 = \infty$

34) $\lim_{h \to 0^+} \frac{\sqrt{0+h}-0}{h}$ $a=0$

$\lim_{h \to 0^+} \frac{\sqrt{h}}{h}$ $\frac{h^{1/2}}{h^1}$ $\frac{\cancel{\sqrt{h}}}{\cancel{\sqrt{h}}\sqrt{h}}$

$\lim_{h \to 0^+} \frac{1}{\sqrt{h}} = \infty$

35) x^2+1 , x^2+2

36)

3.1

RHD

41) $\lim_{h\to 0^+} \frac{(0+h)^2 - 1 - (-1)}{h}$ $\lim_{h\to 0^+} \frac{2(0+h) - 1 - (-1)}{h}$ $a=0$

$\lim_{h\to 0^-} \frac{h^2}{h}$ $\lim_{h\to 0^+} \frac{2h}{h}$

$\lim_{h\to 0^-} h = \infty$ $\lim_{h\to 0^+} 2 = 2$

44) $f(x) = \begin{cases} x^3 & , x \le 1 \\ 3x+k & , x > 1 \end{cases}$ $(1)^3 = 3(1) + k$

$1 = 3 + k$

$f(x) = \begin{cases} x^3 & , x \le 1 \\ 3x-2 & , x > 1 \end{cases}$ $a=1$ $-3 \quad -3$

$-2 = k$

$\lim_{h\to 0^-} \frac{(1+h)^3 - 1}{h}$ $\lim_{h\to 0^+} \frac{3(1+h) - 2 - 1}{h}$

$\lim_{h\to 0^-} \frac{\cancel{1} + 3h + 3h^2 + h^3 - \cancel{1}}{h}$ $\lim_{h\to 0^+} \frac{\cancel{3} + 3h - \cancel{3}}{h}$

$\lim_{h\to 0^-} \frac{\cancel{h}(3 + 3h + h^2)}{\cancel{h}}$ $\lim_{h\to 0^+} \frac{3\cancel{h}}{\cancel{h}}$

3 $\lim_{h\to 0^+} 3 = 3$

3.1

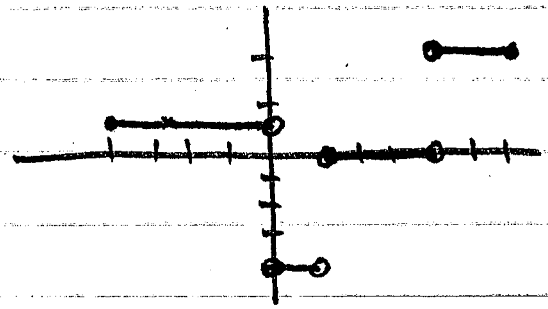
9] $f(x) = 3x - 12$

$$\lim_{h \to 0} \frac{[3(x+h)-12] - [3x-12]}{h}$$

$$\lim_{h \to 0} \frac{[\cancel{3x}+3h-\cancel{12}] - [\cancel{3x}-\cancel{12}]}{h}$$

$$\lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}} = 3$$

26]



21] (a) ~~$\frac{50 \text{ hours}}{30 \text{ days}} = \frac{50}{30}$ $\frac{5}{3} = 1.\overline{6}$~~ $\frac{30}{50} = .6$

7] $f(x) = \sqrt{x+1}$, $a = 3$

$$\lim_{x \to 3} \frac{\sqrt{x+1} - \sqrt{3+1}}{x-3}$$

$$\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2} = \frac{(x+1) - 4}{(x-3)(\sqrt{x+1} + 2)}$$

$$= \frac{\cancel{x-3}}{(\cancel{x-3})(\sqrt{x+1} + 2)}$$

$$= \frac{1}{\sqrt{x+1} + 2}$$

$$\lim_{x \to 3} \frac{1}{\sqrt{x+1} + 2} = \frac{1}{4}$$

3.1

$$\frac{1}{2} - \frac{1}{3}$$

$$\overline{6}$$

5 $f(x) = \frac{1}{x}$, $a = 2$

$$\lim_{x \to 2} \frac{{}^{2}_{2}\frac{1}{x} - \frac{1}{2}{}^{x}_{x}}{x-2}$$

$$\lim_{x \to 2} \frac{\frac{2-x}{2x}}{x-2}$$

$$\lim_{x \to 2} \frac{\overset{-1}{\cancel{2-x}}}{2x} \cdot \frac{1}{\cancel{x-2}}$$

$$\lim_{x \to 2} -\frac{1}{2x} = \boxed{-\frac{1}{4}}$$

1 $f(x) = \frac{1}{x}$, $a = 2$

$$\lim_{h \to 0} \frac{\frac{1}{(2+h)} - \frac{1}{2}}{h}$$

$$\lim_{h \to 0} \frac{2(2+h)}{2(2+h)} \quad \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$$

$$\lim_{h \to 0} \frac{2 - (2+h)}{2h(2+h)}$$

$$\lim_{h \to 0} \frac{-\cancel{h}}{2\cancel{h}(2+h)}$$

$$\lim_{h \to 0} \frac{-1}{2(2+h)} = \boxed{-\frac{1}{4}}$$

3.1

19 $y = x^3$ (a) $m = 3$ $(1,1)$

$\lim_{h\to 0} \frac{(1+h)^3 - 1^3}{h}$

1

1 1

1 2 1

1 3 3 1

$\lim_{h\to 0} \frac{\cancel{1}+3h+3h^2+h^3-\cancel{1}}{h}$

(b) $m = -\frac{1}{3}$ $(1,1)$

$\lim_{h\to 0} \frac{\cancel{h}(3+3h+h^2)}{\cancel{h}}$

$\lim_{h\to 0} 3+3h+h^2 = \boxed{3}$