# 3.6

**43|** $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec^2 t}{2\sec^2 t \tan t}$ $\qquad$ $\sec^2 t - 1$

$= \frac{1}{2\tan t}$ $\qquad$ $(\sec t)^2 - 1$

$2(\sec t)[\sec t \tan t]$

$= \frac{1}{2\tan -\frac{\pi}{4}}$ $\qquad$ $x = \sec^2 \frac{\pi}{4} - 1$

$= -\frac{1}{2}$ $\qquad$ $= 2 - 1 = 1$

$y = \tan -\frac{\pi}{4} = -1$

$y = mx + b$

$-1 = -\frac{1}{2}(1) + b$

$-1 = -\frac{1}{2} + b$

$-\frac{1}{2} = b$

$$\boxed{y = -\frac{1}{2}x - \frac{1}{2}}$$

**35|** $f(u) = \cot \frac{\pi u}{10}$

$f'(u) = -\csc^2\left(\frac{\pi u}{10}\right) \left[\frac{\pi}{10} du\right]$

$f'(x) = -\csc^2\left(\frac{\pi}{10} \cdot 5\sqrt{x}\right)\left[\frac{\pi}{10} \cdot \frac{5}{2} x^{-1/2}\right]$

$u = 5\sqrt{x}$ $\qquad$ $f'(1) = -\csc^2\left(\frac{\pi}{2}\right)\left[\frac{\pi}{4}\right]$

$u = 5x^{1/2}$ $\qquad$ $= -1\left[\frac{\pi}{4}\right]$

$du = \frac{5}{2}x^{-1/2}$ $\qquad$ $= \boxed{\frac{-\pi}{4}}$

# 3.6

**19** $y = \frac{3}{\sqrt{2x+1}}$

$= 3(2x+1)^{-1/2}$

$y' = -\frac{3}{2}(2x+1)^{-3/2}[2]$

$= -3(2x+1)^{-3/2}$

$= \frac{-3}{\sqrt{(2x+1)^3}}$

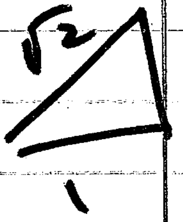
**37** $f(u) = \frac{2u}{u^2+1}$    f   g      $u = 10x^2+x+1$

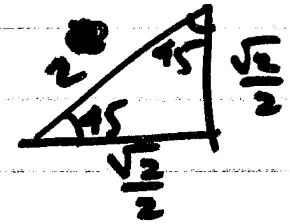
$du = 20x+1$

$f'(u) = \frac{2(u^2+1) - \overbrace{2u\,du\,(2u)}^{4u^2}}{(u^2+1)^2}$

$= \frac{2(20x+1)([10x^2+x+1]^2+1) - \cancel{4}(10x^2+x+1)^2(20x+1)}{[(10x^2+x+1)+1]^2}$

$= \frac{2(1)(1^2+1) - [4\cdot 1^2 \cdot 1]}{(1+1)^2}$




$= \frac{4-4}{2} = \boxed{0}$

**41** $x = 2\cos t$, $y = 2\sin t$    $t = \frac{\pi}{4}$

$x = 2\cos\frac{\pi}{4} = \sqrt{2}$    $y = 2\sin\frac{\pi}{4} = \sqrt{2}$    $(\sqrt{2}, \sqrt{2})$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos t}{-2\sin t} = \frac{2\cos\frac{\pi}{4}}{-2\sin\frac{\pi}{4}} = \frac{\sqrt{2}}{-\sqrt{2}} = -1$

$y = mx+b$

$\sqrt{2} = -\sqrt{2} + b$

$2\sqrt{2} = b$

$\boxed{y = -x + 2\sqrt{2}}$

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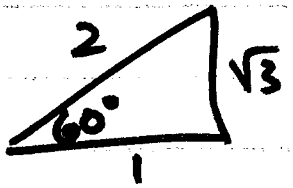
**47** $x = t - \sin t \quad y = 1 - \cos t$




$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin t}{1-\cos t}$$

$$\frac{dy}{dx}\left(\frac{\pi}{3}\right) = \frac{\sin\frac{\pi}{3}}{1-\cos\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{1-\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$x = \frac{\pi}{3} - \sin\frac{\pi}{3} \qquad y = 1 - \cos\frac{\pi}{3} \qquad \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$= \frac{\pi}{3} - \frac{\sqrt{3}}{2} \qquad = 1 - \frac{1}{2} = \frac{1}{2}$$

$$y = mx + b$$

$$\frac{1}{2} = \sqrt{3}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) + b$$

$$\frac{1}{2} - \frac{\sqrt{3}\pi}{3} + \frac{3}{2} = b$$

$$y = \sqrt{3}x + \left(-\frac{\sqrt{3}\pi}{3} + 2\right)$$

**49** $x = t^2 + t, \; y = \sin t$

(a) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{2t+1}$

(b) $\frac{d}{dt}\frac{dy}{dx} = \frac{d}{dt}\left(\frac{\cos t^{\,f}}{2t+1_{\,g}}\right) = \frac{-\sin t\,(2t+1) - 2\cos t}{(2t+1)^2}$

(c) $\frac{d}{dx}\frac{dy}{dx} = \underbrace{\left(\frac{d}{dt}\frac{dy}{dx}\right)}\div\frac{dx}{dt} = \dfrac{\frac{-\sin t\,(2t+1) - 2\cos t}{(2t+1)^2}}{2t+1}$

$$= \frac{-\sin t\,(2t+1) - 2\cos t}{(2t+1)^3}$$

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56 (a) $2f(x)$ @ $x=2$ $\quad$ ~~$2f(2) = 2$~~

$\frac{d}{dx} 2f(x) = 2f'(x)$ @ $x=2$ $\quad 2f'(2) = 2 \cdot \frac{1}{3} = \boxed{\frac{2}{3}}$

(c) $\frac{d}{dx} f(x) \cdot g(x) = f'(x)g(x) + g'(x)f(x)$ @ $x=3$

$f'(3)g(3) + g'(3)f(3)$

$(2\pi)(-4) + (5)(3) = \boxed{-8\pi + 15}$

(e) $\frac{d}{dx} f(g(x)) = f'(g(x))\ g'(x)$ @ $x=2$

$f'(g(2))\ g'(2)$

$f'(2)\ g'(2)$

$(\frac{1}{3})(-3) = \boxed{-1}$

$(2x+4)^2$

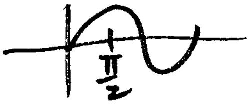
$2(2x+4)$

$(g(x))^2$

$2(g(x))$

(g) $\frac{1}{g^2(x)} = \frac{-2g(x)g'(x)}{(g^2(x))^2} = \frac{-2g(3)g'(3)}{(g^2(3))^2} = \frac{-2(-4)(5)}{(-4)^4}$

$= \frac{40}{256} = \boxed{\frac{5}{32}}$




$\frac{\pi}{4}x$

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**55|**

$y = 2\tan(\pi x/4)$ $\qquad$ $y(1) = 2\tan(\pi(1)/4) = 2$

$y' = 2\sec^2(\pi x/4)\left[\frac{\pi}{4}\right]$ $\qquad$ $(1, 2)$

$y'(1) = 2\sec^2(\pi(1)/4)\left[\frac{\pi}{4}\right]$

$= 2(\sqrt{2})^2\left[\frac{\pi}{4}\right]$

$= \cancel{2}\cdot\cancel{2}\left[\frac{\pi}{\cancel{4}}\right] = \pi$

TANGENT

$y = mx + b$

$2 = \pi(1) + b$

$2 - \pi = b$

$\boxed{y = \pi x + (2 - \pi)}$

NORMAL $\quad m = -\frac{1}{\pi}$

$y = mx + b$

$2 = -\frac{1}{\pi}(1) + b$

$2 + \frac{1}{\pi} = b$

$\boxed{y = -\frac{1}{\pi}x + \left(2 + \frac{1}{\pi}\right)}$

**17|**

$\quad f \qquad g$

$y = \sin^3 x \tan 4x$

$y' = 3\sin^2 x[\cos x]\tan 4x + \sec^2 4x[4]\sin^3 x$

$= 3\sin^2 x \cos x \tan 4x + 4\sec^2 4x \sin^3 x$

$(\sin x)^3$

$3(\sin x)^2$

**35|**

$f(u) = \cot \frac{\pi u}{10}$ $\qquad$ $u = 5x^{1/2} = 5\sqrt{x}$

$f'(u) = -\csc^2\left(\frac{\pi u}{10}\right)\left[\frac{\pi}{10}\,du\right]$ $\qquad$ $du = \frac{5}{2}x^{-1/2}$

$f'(x) = -\csc^2\left(\frac{\pi}{10}\cdot 5\sqrt{x}\right)\left[\frac{\pi}{10}\cdot\frac{5}{2\sqrt{x}}\right]$

$= -\csc^2\left(\frac{\pi}{2}\sqrt{x}\right)\left[\frac{\pi}{4\sqrt{x}}\right]$

$= -\frac{\pi}{4\sqrt{x}}\csc^2\left(\frac{\pi}{2}\sqrt{x}\right)$

$f'(1) = \frac{-\pi}{4\sqrt{1}}\csc^2\left(\frac{\pi}{2}\sqrt{1}\right)$

$= \boxed{-\frac{\pi}{4}}$

3.6

5] $$y = \left(\frac{\overset{f}{\sin x}}{\underset{g}{1+\cos x}}\right)^2$$

$$y' = 2\left(\frac{\sin x}{1+\cos x}\right)^1 \left[\frac{\cos x(1+\cos x) - -\sin x(\sin x)}{(1+\cos x)^2}\right]$$

$$= 2\left(\frac{\sin x}{1+\cos x}\right)\left[\frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}\right]$$

$$= 2\left(\frac{\sin x}{1+\cos x}\right)\left(\frac{\cancel{\cos x+1}}{(1+\cos x)^2}\right) = \boxed{\frac{2\sin x}{(1+\cos x)^3}}$$

$$= \frac{2\sin x}{(1+\cos x)^2}$$

13] $$y = (x + x^{1/2})^{-2}$$

$$y' = -2(x+x^{1/2})^{-3}\left[1+\tfrac{1}{2}x^{-1/2}\right]$$

$$= \frac{-2 - x^{-1/2}}{(x+x^{1/2})^3} = \frac{-2 - \frac{1}{\sqrt{x}}}{(x+\sqrt{x})^3}$$

$$= \frac{-\frac{2\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}}}{(x+\sqrt{x})^3}$$

$$= \frac{\frac{-2\sqrt{x} - 1}{\sqrt{x}}}{(x+\sqrt{x})^3}$$

$$= \frac{-2\sqrt{x} - 1}{\sqrt{x}\,(x+\sqrt{x})^3}$$

3.6

19| $y = \dfrac{3}{\sqrt{2x+1}}$

$y = 3(2x+1)^{-1/2}$

$y' = -\frac{3}{2}(2x+1)^{-3/2}\,[2]$

$= \dfrac{-3}{(2x+1)^{3/2}}$

22| $y = (1+\cos 2x)^2$

$y' = 2(1+\cos 2x)^1\,[-\sin 2x]\,[2]$

$= -4\sin 2x\,(1+\cos 2x)$

32| $y = 9\tan\left(\frac{1}{3}x\right)$ $\qquad$ $\left(\sec\left(\frac{1}{3}x\right)\right)^2$

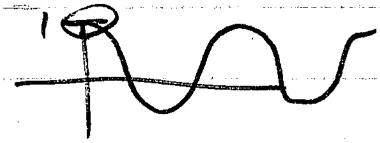
$y' = 9\sec^2\left(\frac{1}{3}x\right)\left[\frac{1}{3}\right]$

$= 3\sec^2\left(\frac{1}{3}x\right)$

$y'' = 6\sec\left(\frac{1}{3}x\right)\left[\sec\left(\frac{1}{3}x\right)\tan\left(\frac{1}{3}x\right)\right]\left[\frac{1}{3}\right]$

$y'' = 2\sec^2\left(\frac{1}{3}x\right)\tan\left(\frac{1}{3}x\right)$

53| $y = \sin\left(\frac{1}{2}x\right)$ $\qquad$ $\frac{1}{2}$




$y' = \underbrace{\cos\left(\frac{1}{2}x\right)}_{1}\left[\frac{1}{2}\right]$

54| $y = \sin mx$ $\qquad$ $(0,0)$ $\qquad$ $\boxed{y = mx}$

$y' = \cos mx\;[m]$

$y'(0) = \cos m(0)\;[m] = m$

3.6

56 (a) $\frac{d}{dx} 2f(x)$

$2f'(x)$

$2f'(2)$

$2\left(\frac{1}{3}\right) = \boxed{\frac{2}{3}}$

(b) $\frac{d}{dx}[f(x)+g(x)]$

$f'(x)+g'(x)$

$f'(3)+g'(3)$

$\boxed{2\pi + 5}$

(c) $\frac{d}{dx}[f(x)\cdot g(x)]$

$f'(x)g(x)+g'(x)f(x)$

$f'(3)g(3)+g'(3)f(3)$

$2\pi(-4) + 5(3)$

$\boxed{-8\pi+15}$

(d) $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x)-g'(x)f(x)}{[g(x)]^2}$

$\frac{f'(2)g(2)-g'(2)f(2)}{[g(2)]^2}$

$\frac{\frac{1}{3}(2)-(-3)(8)}{(2)^2}$

$\frac{\frac{2}{3}+24}{4}$ $\frac{74}{12}$ $\boxed{\frac{37}{6}}$

(e) $\frac{d}{dx} f(g(x)) = f'(g(x))\,g'(x)$

$f'(g(2))\,g'(2)$

$f'(2)\,g'(2)$

$\frac{1}{3}(-3) = \boxed{-1}$

(f) $\frac{d}{dx}(f(x))^{1/2} = \frac{1}{2}(f(x))^{-1/2} f'(x)$

$\frac{1}{2}(f(2))^{-1/2} f'(2)$

$\frac{1}{2}(8)^{-1/2}\left(\frac{1}{3}\right)$

$\frac{1}{2}\ \frac{1}{2\sqrt{2}}\ \frac{1}{3} = \boxed{\frac{1}{12\sqrt{2}}}$ $\frac{\sqrt{2}}{24}$

3.6

36] $f(u) = u + \frac{1}{\cos^2 u}$ $\qquad u = \pi x$

$= u + \cos^{-2} u$ $\qquad du = \pi$

$f'(u) = 1 - 2\cos^{-3} u \; du$

$f'(x) = 1 - 2\cos^{-3}(\pi x)\,[\pi]$

$= 1 - 2\pi\cos^{-3}(\pi x)$