3.7

28 $x^{2/3} + y^{2/3} = 1$

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$

$-\frac{2}{3}x^{-1/3}$

$\frac{\frac{2}{3}y^{-1/3}\frac{dy}{dx}}{\frac{2}{3}y^{-1/3}} = \frac{-\frac{2}{3}x^{-1/3}}{\frac{2}{3}y^{-1/3}}$

$\frac{dy}{dx} = -\frac{x^{-1/3}}{y^{-1/3}} = -\frac{y^{1/3}}{x^{1/3}}$ f g

$$\frac{-\frac{1}{3}y^{-2/3}x^{1/3}\frac{dy}{dx} + \frac{1}{3}x^{-2/3}y^{1/3}}{x^{2/3}} = \frac{d^2y}{dx^2}$$

$$\frac{-\frac{1}{3}y^{-2/3}x^{1/3}\left(-\frac{y^{1/3}}{x^{1/3}}\right) + \frac{1}{3}x^{-2/3}y^{1/3}}{x^{2/3}} =$$

$$\frac{+\frac{1}{3}y^{-1/3} + \frac{1}{3}x^{-2/3}y^{1/3}}{x^{2/3}}$$

$$\frac{y^{-1/3} + x^{-2/3}y^{1/3}}{3x^{2/3}}$$

3.7

57) f g

$\boxed{xy + 2x - y = 0}$

$2x + y = 0$

$y = -2x$

$\perp \quad -2$

$\left(\frac{1}{2}\right)$

$y + x\frac{dy}{dx} + 2 - \frac{dy}{dx} = 0$

$-y \quad -2 \quad -y-2$

$\frac{\frac{dy}{dx}(x-1)}{x-1} = \frac{-y-2}{x-1}$

$\frac{dy}{dx} = \frac{-y-2}{x-1} = \frac{1}{2}$

$-2y - 4 = x - 1$

$+1 \quad +1$

$\boxed{-2y - 3 = x}$

-3

$(-2y-3)y + 2(-2y-3) - y = 0$

$-2y^2 - 3y - 4y - 6 - y = 0$

$-2y^2 - 8y - 6 = 0$

$y^2 + 4y + 3 = 0$

$(y+3)(y+1) = 0$

$y = -3, \quad y = -1$

$(3, -3) \quad (-1, -1)$

TANGENT

$y = mx + b$

$-3 = \left(\frac{1}{2}\right)(3) + b$

$-3 = \frac{3}{2} + b$

$-\frac{9}{2} = b$

$y = \frac{1}{2}x - \frac{9}{2}$

$-1 = \left(\frac{1}{2}\right)(-1) + b$

$-1 = -\frac{1}{2} + b$

$-\frac{1}{2} = b$

$y = \frac{1}{2}x - \frac{1}{2}$

3.7

7) $x + \tan(xy) = 0$

$1 + \sec^2(xy)\left[y + x\frac{dy}{dx}\right] = 0$

$1 + y\sec^2(xy) + x\sec^2(xy)\frac{dy}{dx} = 0$

$-x\sec^2(xy)\frac{dy}{dx} \quad -x\sec^2(xy)\frac{dy}{dx}$

$$\frac{dy}{dx} = -\frac{1 + y\sec^2(xy)}{x\sec^2(xy)} = \frac{-x\sec^2(xy)\frac{dy}{dx}}{-x\sec^2(xy)}$$

$$= \frac{-1}{x\sec^2(xy)} + \frac{-y\sec^2(xy)}{x\sec^2(xy)}$$

$$-\frac{1}{x}\cos(xy) + -\frac{y}{x}$$

f g f g

13) $x^2y + -xy^2 = 4$

$2xy + x^2\frac{dy}{dx} + -y^2 + -2xy\frac{dy}{dx} = 0$

$-2xy \qquad +y^2 \qquad -2xy + y^2$

$$\frac{\frac{dy}{dx}(x^2 - 2xy)}{(x^2 - 2xy)} = \frac{-2xy + y^2}{x^2 - 2xy} = \frac{dy}{dx}$$

$x^2 - 2xy = 0$

$x(x - 2y) = 0$

$x \neq 0 \qquad x \neq 2y$

$\frac{x}{2} \neq y$

3.7

41 $y = 3(\csc x)^{3/2}$

$\frac{dy}{dx} = \frac{9}{2}(\csc x)^{1/2}[-\csc x \cot x]$

$-\frac{9}{2}(\csc x)^{3/2}(\cot x)$

49 $x^2 + xy + y^2 = 7$

$x^2 = 7$

$x = \pm\sqrt{7}$

$(\sqrt{7}, 0), (-\sqrt{7}, 0)$

$(\sqrt{7})^2 + \sqrt{7}y + y^2 = 7$

$7 + \sqrt{7}y + y^2 = 7$

$-7 \quad \sqrt{7}y + y^2 = 0$

$y(\sqrt{7} + y) = 0$

$y = 0 \qquad y = -\sqrt{7}$

$(\sqrt{7}, 0) \qquad (-\sqrt{7}, 0)$

fg

$x^2 + xy + y^2 = 7$

$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$-2x \qquad -y \qquad\qquad -2x - y$

$\frac{dy}{dx}(x + 2y) = -2x - y$

$\frac{\frac{dy}{dx}(x+2y)}{x+2y} = \frac{-2x-y}{x+2y} = \frac{dy}{dx}$

$= \frac{-2\sqrt{7} - 0}{7 + 2(0)}$

$= -2$

$\frac{2\sqrt{7} - 0}{-\sqrt{7} - 2(0)}$

$= -2$

3.7

53) $v = 8\sqrt{s-t} + 1 \qquad 8(s-t)^{1/2} + 1$

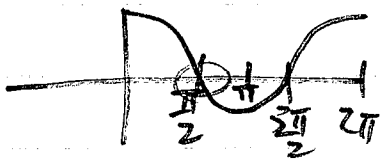
$\frac{dv}{dt} = 4(s-t)^{-1/2}\left[\frac{ds}{dt} - 1\right]$

$= 4(s-t)^{-1/2}\left[8\sqrt{s-t} + 1 - 1\right]$

$= 32$

3.7

23) $\overset{f}{\underset{\sim}{2xy}} + \overset{g}{\pi \sin y} = 2\pi$, $(1, \frac{\pi}{2})$



$2y + 2x\frac{dy}{dx} + \pi \cos y \frac{dy}{dx} = 0$

$-2y$ $\qquad -2y$

$\frac{\frac{dy}{dx}(2x + \pi \cos y)}{(2x + \pi \cos y)} = \frac{-2y}{2x + \pi \cos y} = \frac{dy}{dx}$

$\frac{-2(\frac{\pi}{2})}{2(1) + \pi \cos \frac{\pi}{2}} = \frac{-\pi}{2 + 0} = -\frac{\pi}{2}$

$y = mx + b$

$\frac{\pi}{2} = -\frac{\pi}{2}(1) + b$

$\pi = b$

TANGENT $y = -\frac{\pi}{2}x + \pi$

$y = mx + b$

$\frac{\pi}{2} = \frac{2}{\pi}(1) + b$

$\frac{\pi}{2} - \frac{2}{\pi} = b$

NORMAL $y = \frac{2}{\pi}x + (\frac{\pi}{2} - \frac{2}{\pi})$

41) $y = 3(\csc x)^{3/2}$

$\frac{dy}{dx} = \frac{9}{2}(\csc x)^{1/2}[-\csc x \cot x]$

$= \frac{-9}{2}(\csc x)^{3/2} \cot x$

$x^{1/2} \cdot x$

$x^{3/2}$

47) $\overset{f}{x^3}\overset{g}{y^2} = \cos(\pi y)$ $\qquad (-1, 1)$

(a) $(-1)^3(1)^2 = \cos(\pi(1))$

$-1 = -1$ ✓

(b) $3x^2y^2 + 2x^3y\frac{dy}{dx} = -\sin(\pi y)[\pi \frac{dy}{dx}]$

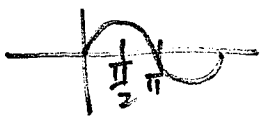
$-2x^3y\frac{dy}{dx}$ $\qquad -2x^3y\frac{dy}{dx}$

$3x^2y^2 = \frac{dy}{dx}(-\pi \sin(\pi y) - 2x^3y)$

$3x^2y^2$ $\qquad (-\pi \sin(\pi y) - 2x^3y)$

3.7

$$\frac{dy}{dx} = \frac{3x^2y^2}{-\pi\sin(\pi y) - 2x^3y}$$



$(-1,1)$

$$= \frac{3(-1)^2(1)^2}{-\pi\sin(\pi(1)) - 2(-1)^3(1)} = \boxed{\frac{3}{2}}$$

53

$$v = 8(s-t)^{1/2} + 1 = \frac{ds}{dt}$$

$$\frac{dv}{dt} = 4(s-t)^{-1/2}\left[\frac{ds}{dt} - 1\right]$$

$$= 4(s-t)^{-1/2}\left[8(s-t)^{1/2} + 1 - 1\right]$$

$$= \frac{32(s-t)^{1/2}}{(s-t)^{1/2}}$$

$$= 32 \text{ ft/sec}^2$$

57

$2x + y = 0$

$y = -2x$

$m = -2$

f & g

$xy + 2x - y = 0$

$y + x\frac{dy}{dx} + 2 - \frac{dy}{dx} = 0$

$-y \quad -2 \quad -y - 2$

$\frac{dy}{dx}(x-1) = -y - 2$

$\frac{dy}{dx} = \frac{-y-2}{x-1}$

$(-2y-3)y + 2(-2y-3) - y = 0(x-1)$

$-2y^2 - 3y - 4y - 6 - y = 0$

$-2y^2 - 8y - 6 = 0$

$y^2 + 4y + 3 = 0$

$(y+3)(y+1) = 0$

$y = -3 \quad y = -1$

$(3,-3) \quad (-1,-1) \quad m = -2$

$\frac{-y-2}{x-1} = \frac{1}{2}$

$-2y - 4 = x - 1$

$+1 \quad +1$

$-2y - 3 = x$

3.7

$y = mx + b$ $\quad m = -2, \quad (3, -3)$

$-3 = -2(3) + b$

$3 = b$

$\boxed{y = -2x + 3}$

$(-1, -1)$

$-1 = -2(-1) + b$

$-3 = b$

$\boxed{y = -2x - 3}$

f g
xy^2

$f = x$
$f' = 1$
$g = y^2$
$g' = 2y\frac{dy}{dx}$
$f'g$
$1y^2$

3.7

f g f g g'
1 $x^2y + xy^2 = 6$

$2xy + x^2\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} = 0$

$-2xy \quad -y^2 \quad -2xy - y^2$

$\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^2$

$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$

f
3 $y^2 = \frac{x-1}{x+1}$
g

$2y\frac{dy}{dx} = \frac{(x+1) - (x-1)}{(x+1)^2}$

$2y\frac{dy}{dx} = \frac{2}{(x+1)^2}$

$\frac{2y}{2y}\frac{dy}{dx} = \frac{2}{2y(x+1)^2}$

$\frac{dy}{dx} = \frac{1}{y(x+1)^2}$

37 $y = x\sqrt{x^2+1}$

f g
$= x(x^2+1)^{1/2}$

$\frac{dy}{dx} = (x^2+1)^{1/2} + \frac{1}{2}(x^2+1)^{-\frac{1}{2}}[2x]\cdot x$

$= (x^2+1)^{1/2} + x^2(x^2+1)^{-\frac{1}{2}}$

3.7

24 $x \sin 2y = y \cos 2x$

$\sin 2y + \cos 2y \left[2\frac{dy}{dx}\right] \cdot x = \frac{dy}{dx}\cos 2x + -\sin 2x\,[2]\,y$

$-\sin 2y \qquad -\frac{dy}{dx}\cos 2x \qquad -\frac{dy}{dx}\cos 2x \qquad -\sin 2y$

$$\frac{dy}{dx}\left[2x\cos 2y - \cos 2x\right] = \frac{-\sin 2x - \sin 2y}{2x\cos 2y - \cos 2x} = \frac{dy}{dx}$$

$$\left[2x\cos 2y - \cos 2x\right]$$

28 $x^{2/3} + y^{2/3} = 1$

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$

$-\frac{2}{3}x^{-1/3} \qquad -\frac{2}{3}x^{-1/3}$

$\frac{2}{3}y^{-1/3}\frac{dy}{dx} = -\frac{2}{3}x^{-1/3}$

$\frac{2}{3}y^{-1/3} \qquad \frac{2}{3}y^{-1/3}$

$\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$ ← (f/g)

$$\frac{d^2y}{dx^2} = -\left(\frac{\frac{1}{3}y^{-2/3}\frac{dy}{dx}\cdot x^{1/3} - \frac{1}{3}x^{-2/3}y^{1/3}}{(x^{1/3})^2}\right)$$

$$= -\left(\frac{\frac{1}{3}y^{-2/3}\left(\frac{y^{1/3}}{x^{1/3}}\right)x^{1/3} - \frac{1}{3}x^{-2/3}y^{1/3}}{x^{4/3}}\right)$$

$$= -\left(\frac{\frac{1}{3}y^{-1/3} - \frac{1}{3}x^{-2/3}y^{1/3}}{x^{2/3}}\right)$$

$$= -\left(\frac{x^{2/3}\frac{1}{3y^{1/3}} - \frac{y^{1/3}}{3x^{2/3}}\cdot x^{1/3}}{x^{2/3}}\right)$$

$$= -\left(\frac{\frac{x^{2/3} - y^{2/3}}{3x^{2/3}y^{1/3}}}{x^{2/3}}\right) = \boxed{-\frac{x^{2/3} - y^{2/3}}{3x^{4/3}y^{1/3}}}$$