3.8

15] $y = \csc^{-1}(x^2+1)$

$$y' = \frac{-1}{|x^2+1|\sqrt{(x^2+1)^2-1}}\,[2x]$$

$$= \frac{-2x}{x^2+1\sqrt{x^4+2x^2+\cancel{1}-\cancel{1}}}$$

$$= \frac{-2x}{x^2+1\sqrt{x^2(x^2+2)}}$$

$$= \frac{-2\cancel{x}}{(x^2+1)\cancel{|x|}\sqrt{x^2+2}}$$

$$= \boxed{\frac{-2}{(x^2+1)\sqrt{x^2+2}}}$$

21] $y = \tan^{-1}\sqrt{x^2-1} + \csc^{-1} x$

$$y' = \frac{1}{1+(\sqrt{x^2-1})^2}\left[\frac{x}{\sqrt{x^2-1}}\right] + \frac{-1}{|x|\sqrt{x^2-1}}$$

$$= \cancel{\frac{1}{1+x^2}}$$

$$= \frac{1}{\cancel{1}+x^2-\cancel{1}}\left[\frac{x}{\sqrt{x^2-1}}\right] + \frac{-1}{|x|\sqrt{x^2-1}}$$

$$= \frac{\cancel{x}}{x^{\cancel{2}}\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}}$$

$$= \frac{1}{x\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}} = 0$$

$(x^2-1)^{1/2}$

$\cancel{\tfrac{1}{2}}(x^2-1)^{-1/2}[\cancel{2}x]$

$\frac{x}{\sqrt{x^2-1}}$

17] $y = \sec^{-1}\frac{1}{t}$

3.8

17 $y = \sec^{-1} \frac{1}{t}$

$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$

$(\cos^{-1} x)' = -(\sin^{-1} x)'$

$$y' = \frac{1}{|\frac{1}{t}|\sqrt{(\frac{1}{t})^2 - 1}}$$

$$y' = \frac{1}{|\frac{1}{t}|\sqrt{(\frac{1}{t})^2 - 1}}\left[\frac{-1}{t^2}\right]$$

$$y' = \frac{1}{t^2|\frac{1}{t}|\sqrt{\frac{1}{t^2} - 1}}$$

$$y' = \frac{-|t|}{\sqrt{t^4}\sqrt{\frac{1}{t^2} - 1}}$$

$$y' = \frac{-|t|}{\sqrt{t^4(\frac{1}{t^2} - 1)}}$$

$$y' = \frac{-|t|}{\sqrt{t^2 - t^4}}$$

$$y' = \frac{-|t|}{\sqrt{t^2(1 - t^2)}}$$

$$y' = \frac{-\cancel{|t|}}{\cancel{|t|}\sqrt{1 - t^2}}$$

$$y' = \frac{-1}{\sqrt{1 - t^2}}$$

3.8

29| (a) $f(x) = \cos x + 3x$

$f'(x) = -\sin x + 3$

(b) $f(0) = 1 \quad f'(0) = 3$

(c) $f^{-1}(1) = 0 \quad (f^{-1})'(1) = \frac{1}{3}$

$y = 3x$ $\quad$ $y = \frac{1}{3}x$

$y' = 3$ $\quad$ $y' = \frac{1}{3}$

7| $y = \overset{f}{x} \sin^{-1} \overset{g}{x} + \sqrt{1-x^2}$

$y' = \underbrace{\sin^{-1}x}_{f'g} + \underbrace{\frac{1}{\sqrt{1-x^2}} \cdot x}_{g'f} + \underbrace{\frac{1}{2}(1-x^2)^{-1/2}[-2x]}$

$y' = \sin^{-1}x + \cancel{\frac{x}{\sqrt{1-x^2}}} + \cancel{\frac{-x}{\sqrt{1-x^2}}}$

$\boxed{y' = \sin^{-1} x}$

x $\quad$ y

$y = x^2$ $\quad$ $y = \sqrt{x}$

$y = 2x$ $\quad$ $y = \frac{1}{2}x^{-1/2}$

$y = 2$ $\quad$ $y = \frac{1}{2}$

$(x, y) \to (y, x)$

$(1-x^2)^{1/2}$

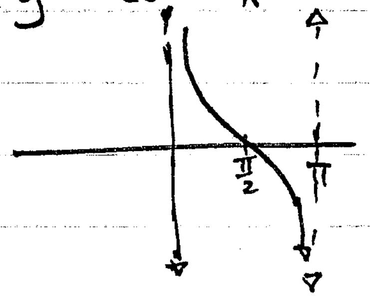
31| $x = \arctan \overset{f}{t}$

$x' = \frac{1}{1+t^2_{g}} > 0$

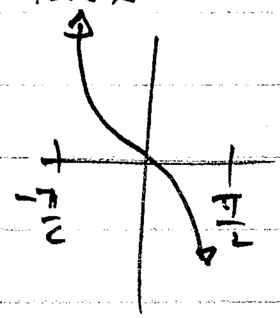
$x'' = \frac{0(1+t^2) - 2t(1)}{(1+t^2)^2}$

$= \frac{-2t}{(1+t^2)^2}, \; t \geq 0 \; \leq 0$

$y = \cot x$

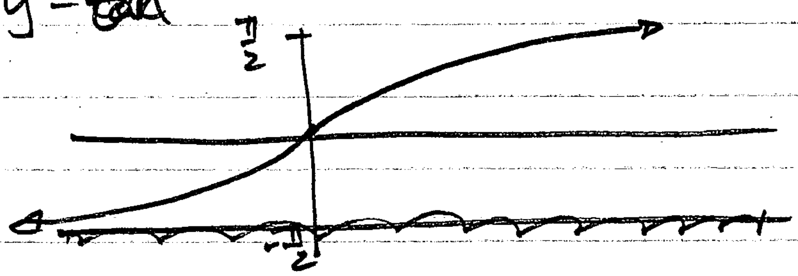


$y = \tan x$



LIMIT $= \frac{\pi}{2}$

$y = \cancel{\tan}^{-x}$



3.8

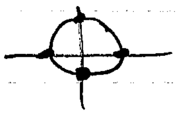
21| $y=\overbrace{\tan^{-1}\sqrt{x^2-1}}+\csc^{-1}x$

$y'=\frac{1}{1+(\sqrt{x^2-1})^2}\left[\cancel{\tfrac{1}{2}}(x^2-1)^{-\frac{1}{2}}[\cancel{2}x]\right]+\frac{-1}{|x|\sqrt{x^2-1}}$

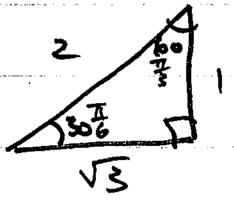
$y'=\frac{1}{1+(x^2-1)}\left[\frac{x}{\sqrt{x^2-1}}\right]-\frac{1}{|x|\sqrt{x^2-1}}$

$y'=\frac{1}{x^{\cancel{2}}}\left[\frac{\cancel{x}}{\sqrt{x^2-1}}\right]-\frac{1}{|x|\sqrt{x^2-1}}$

$=\frac{1}{x\sqrt{x^2-1}}-\frac{1}{|x|\sqrt{x^2-1}}=0\ ,\ x>1$



23| $y=\sec^{-1}x\ ,\quad x=\frac{2}{1}$

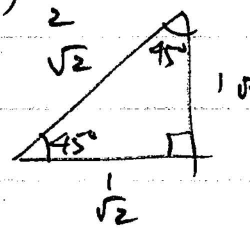


$\sec=\frac{hyp}{adj}$

$60\frac{\pi}{180}$

$y'=\frac{1}{|x|\sqrt{x^2-1}}$

$\left(2,\ \frac{\pi}{3}\right)$



$y'=\frac{1}{2\sqrt{2^2-1}}=\frac{1}{2\sqrt{3}}$

$y=mx+b$

$\frac{\pi}{3}=\frac{1}{\cancel{2}\sqrt{3}}(\cancel{2})+b$

$\frac{\pi}{3}-\frac{1}{\sqrt{3}}=b$

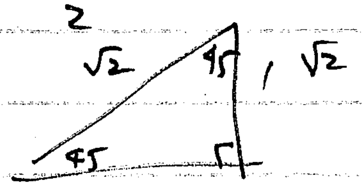
$y=\frac{1}{2\sqrt{3}}x+\left(\frac{\pi}{3}-\frac{1}{\sqrt{3}}\right)$

3.8

7

$$y = \underset{f}{x} \sin^{-1} \underset{g}{x} + \sqrt{1-x^2}$$
$$= x\sin^{-1}x + (1-x^2)^{\frac{1}{2}}$$
$$y' = \sin^{-1}x + \frac{1}{\sqrt{1-x^2}} \cdot x + \frac{1}{\cancel{2}}(1-x^2)^{-\frac{1}{2}}\,[-\cancel{2}x]$$
$$= \sin^{-1}x + \cancel{\frac{x}{\sqrt{1-x^2}}} - \cancel{\frac{x}{\sqrt{1-x^2}}}$$
$$= \sin^{-1}x$$

27 (a) $y = \tan x \qquad (\frac{\pi}{4}, 1)$



$$y' = \sec^2 x$$
$$y'(\tfrac{\pi}{4}) = \sec^2(\tfrac{\pi}{4}) = (\sqrt{2})^2 = 2$$

$\frac{1}{\sqrt{2}} \qquad \frac{2}{\sqrt{2}} = \sqrt{2}$

$\frac{\sqrt{2}}{1} = \sqrt{2}$

$$y = mx + b$$
$$1 = 2(\tfrac{\pi}{4}) + b$$
$$1 - \tfrac{\pi}{2} = b$$
$$\boxed{y = 2x + (1 - \tfrac{\pi}{2})}$$

(b) $y = \tan^{-1}x \qquad (1, \frac{\pi}{4})$

$m = \frac{1}{2} \qquad y = mx + b$

$$\frac{\pi}{4} = \frac{1}{2}(1) + b$$
$$\frac{\pi}{4} - \frac{1}{2} = b$$
$$\boxed{y = \tfrac{1}{2}x + (\tfrac{\pi}{4} - \tfrac{1}{2})}$$

3.8

9 $x(t) = \sin^{-1}\left(\frac{t}{4}\right)$ $t=3$

$\frac{1}{4}t$ $\frac{t}{4}\frac{f}{g}$ $\frac{1\cdot 4 - 0\cdot t}{4^2}$

$\frac{4}{16} = \frac{1}{4}$

$x'(t) = \frac{1}{\sqrt{1-\left(\frac{t}{4}\right)^2}}\left[\frac{1}{4}\right]$

$= \frac{1}{\sqrt{1-\left(\frac{3}{4}\right)^2}}\left[\frac{1}{4}\right]$

$= \frac{1}{\sqrt{1-\frac{9}{16}}}\left[\frac{1}{4}\right]$

$= \frac{1}{\sqrt{\frac{7}{16}}}\left[\frac{1}{4}\right]$

$= \frac{1}{\frac{\sqrt{7}}{4}}\left[\frac{1}{4}\right]$

$= \frac{\cancel{4}}{\sqrt{7}}\cdot\frac{1}{\cancel{4}} = \frac{1}{\sqrt{7}}$

10 $\frac{1}{4}t^{\frac{1}{2}}$

$\frac{1}{8}t^{-\frac{1}{2}}$

3.8

6. $y = s\sqrt{1-s^2} + \cos^{-1} s$

$= \overset{f}{s}\,(1-s^2)^{\frac{1}{2}\,g} + \cos^{-1} s$

$y' = (1-s^2)^{\frac{1}{2}} + \frac{1}{\cancel{2}}(1-s^2)^{-\frac{1}{2}}[-\cancel{2}s]\cdot s + \frac{-1}{\sqrt{1-s^2}}$

$= \underline{(1-s^2)^{\frac{1}{2}}} - \frac{s^2}{(1-s^2)^{\frac{1}{2}}} - \frac{1}{(1-s^2)^{\frac{1}{2}}}$

$= \frac{\cancel{1} - s^2 - s^2 - \cancel{1}}{(1-s^2)^{\frac{1}{2}}} = \boxed{\frac{-2s^2}{(1-s^2)^{\frac{1}{2}}}}$

3.8

10. $x(t) = \sin^{-1}\left(\frac{\sqrt{t}}{4}\right)$

$= \sin^{-1}\left(\frac{t^{1/2}}{4}\right)$

$= \sin^{-1}\left(\frac{1}{4}t^{1/2}\right)$

$$x'(t) = \frac{1}{\sqrt{1-}} = \frac{1}{\sqrt{1-\left(\frac{1}{4}t^{1/2}\right)^2}}\left[\frac{1}{8}t^{-1/2}\right]$$

$$= \frac{1}{\sqrt{1-\frac{1}{16}t}} \cdot \frac{1}{8t^{1/2}}$$

$$= \frac{1}{8t^{1/2}\sqrt{\frac{1}{16}(16-t)}}$$

$$= \frac{1}{8t^{1/2}\sqrt{\frac{1}{16}}\sqrt{16-t}}$$

$$= \frac{1}{8t^{1/2}\cdot\frac{1}{4}\sqrt{16-t}}$$

$$= \frac{1}{2t^{1/2}\sqrt{16-t}}$$

3.8

13) $y = \sec^{-1}(2s+1)$

2 $y' = \dfrac{1}{|2s+1|\sqrt{(2s+1)^2 - 1}}[2]$

$= \dfrac{2}{|2s+1|\sqrt{4s^2+4s+1-1}}$

$= \dfrac{2}{|2s+1|\sqrt{4s^2+4s}}$

$= \dfrac{2}{|2s+1|\sqrt{4(s^2+s)}}$

$= \dfrac{\cancel{2}}{|2s+1|\cancel{\sqrt{4}}\sqrt{s^2+s}}$

1 $= \dfrac{1}{|2s+1|\sqrt{s^2+s}}$

25) $y = \sin^{-1}\left(\frac{x}{4}\right)$, $x=3$ $\qquad y = \sin^{-1}\left(\frac{3}{4}\right) \approx .848$

$y' = \dfrac{1}{\sqrt{1-\left(\frac{x}{4}\right)^2}}\left[\frac{1}{4}\right]$

$y'(3) = \dfrac{1}{\sqrt{1-\left(\frac{3}{4}\right)^2}}\left[\frac{1}{4}\right]$

$= \dfrac{1}{\sqrt{\frac{7}{16}}}\left[\frac{1}{4}\right]$

$= \dfrac{1}{\frac{\sqrt{7}}{\sqrt{16}}}\left[\frac{1}{4}\right]$

3.8

2 Second $\dfrac{1}{\frac{\sqrt{7}}{4}}$ $[\frac{1}{4}]$

$\frac{\cancel{4}}{\sqrt{7}} \cdot \frac{1}{\cancel{4}} = \frac{1}{\sqrt{7}} = m$ $(3, .848)$

$y = mx + b$

$.848 = \frac{1}{\sqrt{7}}(3) + b$

$-\frac{1}{\sqrt{7}}(3)$ $-\frac{1}{\sqrt{7}}(3)$

$-.286 = b$

$y = \frac{1}{\sqrt{7}}x - .286$

$(0,0) \to (0,0)$

Ex FIND $f^{-1}(0)$ AND $(f^{-1})'(0)$ FOR $f(x) = \sin x + x$

$f^{-1}(0) = 0$ $(f^{-1})'(0) = \frac{1}{2}$

$f(0) = \sin 0 + 0$

$f(0) = 0$

$(7,3) \to (3,7)$

$f'(x) = \cos x + 1$

$f'(0) = \cos(0) + 1 = 2$