3.9

45) $\ln y = \ln \sqrt[5]{\dfrac{(x-3)^4 (x^2+1)}{(2x+5)^3}}$

$$\ln y = \frac{1}{5} \ln \frac{(x-3)^4 (x^2+1)}{(2x+5)^3}$$

$$\ln y = \frac{1}{5}\left[\ln (x-3)^4 + \ln (x^2+1) - \ln (2x+5)^3\right]$$

$$\ln y = \frac{1}{5}\left[4\ln (x-3) + \ln (x^2+1) - 3\ln (2x+5)\right]$$

$$y \cdot \frac{1}{y}\frac{dy}{dx} = \frac{1}{5}\left[\frac{4}{x-3} + \frac{2x}{x^2+1} - \frac{6}{2x+5}\right] \cdot y$$

$$\boxed{\frac{dy}{dx} = \frac{1}{5}\left[\frac{4}{x-3} + \frac{2x}{x^2+1} - \frac{6}{2x+5}\right] \sqrt[5]{\frac{(x-3)^4 (x^2+1)}{(2x+5)^3}}}$$

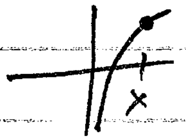
31) $y = \ln(2x)$

GIVEN POINT OF TANGENCY

line $(0,0)$, $(x, \ln(2x))$

$$y' = \frac{1}{2x}[2] = \frac{1}{x}$$

$$m = \frac{\ln(2x) - 0}{x - 0} = \frac{\ln(2x)}{x}$$



$$\frac{1}{x} = \frac{\ln(2x)}{x}$$

$$\frac{1}{x} = \frac{1}{\frac{e}{2}} = \boxed{\frac{2}{e}}$$

$$1 = \ln(2x)$$

$$e^1 = e^{\ln(2x)}$$

$$e = 2x$$

$$\frac{e}{2} = x$$

3.9

49| $y = e^x$ $\quad \boxed{(0,0)}, (x, e^x)$ $\quad m = \frac{e^x - 0}{x - 0} = \frac{e^x}{x}$

$y' = e^x$

$e^x = \frac{e^x}{x}$ $\quad (1, e)$

$x = 1$ $\quad m = e$

$\boxed{y = ex}$

$\frac{e^x}{e^x} = \frac{\frac{e^x}{x}}{e^x}$

$1 = \frac{1}{x}$

$x = 1$

51| (b) $P(t) = \frac{\overset{f}{300}}{\underset{g}{1 + 2^{4-t}}}$

$P'(t) = \frac{-2^{4-t}[-1]\ln 2(300)}{(1+2^{4-t})^2}$

$P'(t) = \boxed{\frac{+2^{4-4}\ln 2(300)}{(1+2^{4-4})^2}} = \frac{\ln 2(300)}{4}$

25| $y = \ln 2 \cdot \log_2 x$ $\quad \overset{f}{\ln 2} \cdot \overset{g}{\log_2 x}$

$y' = \cancel{\ln 2}\frac{1}{x(\cancel{\ln 2})} = \frac{1}{x}, x > 0$ $\quad y' = \underbrace{0(\log_2 x)} + \frac{1}{x(\ln 2)} \cdot \ln 2$

$\frac{d}{dx} \overset{f}{3}\overset{g}{x^2}$

3.9

29) $y = 3^x + 1$

$y' = 3^x(\ln 3) = 5$

$y = 5x - 1 \quad m = 5$

$y = (\ln 3)x - 1$

$3^x (\ln 3) = (\ln 3)$

$3^x = 1$

$x = 0 \qquad (0, 2)$

3.9

29| $y = 3^x + 1$ // $y = 5x - 1$ $y = (\ln 3)x - 1$

$y' = 3^x \ln 3 = 5$ $m = 5$

$3^x \ln 3 = \ln 3$

$3^x = 1$

$x = 0$

19| $y = \ln(\ln x)$

$y' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$

43| $y = (\sin x)^x$

$\ln y = \ln (\sin x)^x$

$\ln y = (\underset{f}{x})(\underset{g}{\ln(\sin x)})$

$y \cdot \frac{1}{y}\frac{dy}{dx} = \left[\ln(\sin x) + \frac{1}{\sin x}[\cos x] \cdot x\right] y$

$\frac{dy}{dx} = \left[\ln(\sin x) + \frac{\cos x}{\sin x} \cdot x\right](\sin x)^x$

$= \left[\ln(\sin x) + x \cot x\right](\sin x)^x$

9| $y = e^{\sqrt{x}}$

$y = e^{x^{1/2}}$

$y' = e^{x^{1/2}}\left[\frac{1}{2}x^{-\frac{1}{2}}\right]$

$= \boxed{\frac{e^{x^{1/2}}}{2\sqrt{x}}}$

3.9

22 $y = \log_5 \sqrt{x}$

$y = \log_5 x^{1/2}$

$y' = \dfrac{1}{x^{1/2}(\ln 5)}\left[\frac{1}{2}x^{-\frac{1}{2}}\right]$

$= \dfrac{1}{x^{1/2}(\ln 5)} \cdot \dfrac{1}{2x^{\frac{1}{2}}} = \dfrac{1}{2(\ln 5)\,x}$

24 $y = \dfrac{1^{\,f}}{\log_2 x_{\,g}}$

$y' = \dfrac{0 \log_2 x - \frac{1}{x(\ln 2)}}{(\log_2 x)^2}$

$y' = \dfrac{-1}{x(\log_2 x)^2(\ln 2)}$

3x

3

26 $y = \log_3(1 + x\ln 3)$

$y' = \dfrac{1}{(1+x\ln 3)\cancel{(\ln 3)}}\left[\cancel{\ln 3}\right]$

$y' = \dfrac{1}{1 + x\ln 3}$

33 $y = x^{\pi}$

$y' = \pi x^{\pi - 1}$

3.9

31) $y = mx + b$

$0 = m(0) + b$

$0 = b$

✓ $y = mx$

$\frac{y}{x} = m$

$y = \ln(2x)$

$y' = \frac{1}{2x}[2] = \frac{1}{x} = m$

$\frac{y}{x} = \frac{1}{x}$

$y = 1$

$\ln(2x) = 1$

$2x = e$

$x = \frac{e}{2}$

$m = \frac{1}{x}$

$= \frac{1}{\frac{e}{2}}$

$= \boxed{\frac{2}{e}}$

37) $f(x) = \ln(x+2)$

$f'(x) = \frac{1}{x+2} \longrightarrow x > -2$

45) $y = \sqrt[5]{\frac{(x-3)^4 (x^2+1)}{(2x+5)^3}}$

$\ln y = \ln \sqrt[5]{\frac{(x-3)^4 (x^2+1)}{(2x+5)^3}}$

$\ln y = \frac{1}{5} \ln \left(\frac{(x-3)^4 (x^2+1)}{(2x+5)^3} \right)$

$\ln y = \frac{1}{5}\left[\ln (x-3)^4 (x^2+1) - \ln (2x+5)^3\right]$

$\ln y = \frac{1}{5}\left[\ln (x-3)^4 + \ln (x^2+1) - \ln (2x+5)^3\right]$

$\ln y = \frac{1}{5}\left[4 \ln (x-3) + \ln (x^2+1) - 3 \ln (2x+5)\right]$

$y \cdot \frac{1}{y}\frac{dy}{dx} = \frac{1}{5}\left[\frac{4}{x-3} + \frac{2x}{x^2+1} - \frac{3}{2x+5}[2]\right] \cdot y$

$\frac{dy}{dx} = \frac{1}{5}\left[\frac{4}{x-3} + \frac{2x}{x^2+1} - \frac{6}{2x+5}\right] \sqrt[5]{\frac{(x-3)^4 (x^2+1)}{(2x+5)^3}}$

3.9

49) $y = e^x \quad , (0,0) \rightarrow y = mx$

$y' = e^x = m \qquad \frac{y}{x} \quad m \qquad e^x = \frac{y}{x}$

$e^x = \frac{e^x}{x}$

$y'(1) = e^1 = e = m \qquad 1 = x$

$\boxed{y = ex}$

3.9

44) $y = x^{\tan x}$

$\ln y = \ln x^{\tan x}$

$\ln y = \overset{f}{\tan x}\ \ln \overset{g}{x}$

$y \cdot \frac{1}{y}\frac{dy}{dx} = \left[\sec^2 x \ln x + \frac{1}{x}\tan x\right] \cdot y$

$\frac{dy}{dx} = \left[\sec^2 x \ln x + \frac{\tan x}{x}\right] x^{\tan x}$

43) $y = (\sin x)^x$

$\ln y = \ln(\sin x)^x = \overset{f}{x}\ln \overset{g}{(\sin x)}$

$y \cdot \frac{1}{y}\frac{dy}{dx} = \left[\ln(\sin x) + \frac{1}{\sin x}[\cos x]\, x\right] y$

$\frac{dy}{dx} = \left[\ln(\sin x) + x\cot x\right](\sin x)^x$

51) $P(t) = \frac{300}{1+2^{4-t}} = \underline{300(1+2^{4-t})^{-1}}$

$P(0) = \frac{300}{1+2^{4-0}} = \frac{300}{17} \approx 18$ students

$P'(t) = -300(1+2^{4-t})^{-2}\left[2^{4-t}\ln 2\,[-1]\right]$

$P'(4) = -300(1+2^{4-4})^{-2}\left[2^{4-4}\ln 2\,[-1]\right]$

$= +300 \cdot \frac{1}{4} \cdot \ln 2 \cdot [+1]$

$= \frac{300\ln 2}{4} = 75\ln 2 \approx 52$ students/day