4.3

5) $y = x\sqrt{8-x^2}$ $\quad [-\sqrt{8}, \sqrt{8}]$

$f \quad g$

$y' = \sqrt{8-x^2} + \frac{1}{2}(8-x^2)^{-\frac{1}{2}}[-2x]\,x = 0 \qquad -2, 2$

DECREASING	INCREASING	DECREASING
$(-\sqrt{8}, -2)$	$(-2, 2)$	$(2, \sqrt{8})$
TEST POINT $x = -2.5$	TEST POINT $x = 0$	TEST POINT $x = 2.5$

$-2 \qquad 2$

SINCE $f(x)$ IS DECREASING ON $(-\sqrt{8}, -2)$ AND SWITCHES TO INCREASING ON $(-2, 2)$, WE HAVE A LOCAL MIN AT $x = -2$ $(-2, -4)$. SINCE $f(x)$ IS ~~DECREASING~~ INCREASING ON $(-2, 2)$ AND SWITCHES TO ~~INCREASING~~ DECREASING ON $(2, \sqrt{8})$, WE HAVE A LOCAL MAX AT $x = 2$ $(2, 4)$.

9) $y = 2x^{1/5} + 3$

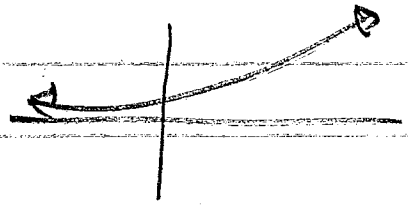
$y' = \frac{2}{5}x^{-4/5}$

$y'' = \frac{-8}{25}x^{-9/5} = \frac{-8}{25x^{9/5}}$

CONCAVE UP	CONCAVE DOWN
$(-\infty, 0)$	$(0, \infty)$
TEST POINT $x = -1$	TEST POINT $x = 1$

0

SINCE y IS CONCAVE UP ON $(-\infty, 0)$ BUT SWITCHES TO CONCAVE DOWN ON $(0, \infty)$, WE HAVE AN INFLECTION POINT $(0, 3)$.



$e^x = 0$

NEVER

4.3

13| $y = xe^x$ (f, g)

$y' = e^x + xe^x$

$y'' = e^x + e^x + xe^x$ 1 or 2

$= 2e^x + xe^x = 0$

$e^x(2+x) = 0$

$e^x = 0$ $\quad 2+x=0$

$x = -2$

CONCAVE DOWN	CONCAVE UP
$(-\infty, -2)$	$(-2, \infty)$
TEST POINT $x=-3$	TEST POINT $x=0$
-2	

1 or 2

INFLECTION PT. $(-2, -\frac{2}{e^2})$

17| $y = x^{1/3}(x-4)$

$= x^{4/3} - 4x^{1/3}$

$y' = \frac{4}{3}x^{1/3} - \frac{4}{3}x^{-2/3}$

$y'' = \frac{4}{9}x^{-2/3} + \frac{8}{9}x^{-5/3}$

$\frac{4}{9x^{2/3}} + \frac{8}{9x^{5/3}} = 0$

$9x^{5/3}\frac{4}{9x^{2/3}} = -\frac{8}{9x^{5/3}}\,9x^{5/3}$

$\frac{4x}{4} = -\frac{8}{4}$

$x = -2$

0, -2

CONCAVE UP	CONCAVE DOWN	CONCAVE UP
$(-\infty, -2)$	$(-2, 0)$	$(0, \infty)$
TEST POINT $x=-3$	TEST POINT $x=-1$	TEST POINT 1
-2	0	

$(-2, -6\sqrt[3]{-2})$

$(-2, 6\sqrt[3]{2}), (0, 0)$

19| $y = \frac{x^3 - 2x^2 + x - 1}{x-2}$ (f, g)

$y' = \frac{(3x^2-4x+1)(x-2) - (x^3-2x^2+x-1)}{(x-2)^2}$

$y' = \frac{3x^3 - 4x^2 + x - 6x^2 + 8x - 2 - x^3 + 2x^2 - x + 1}{(x-2)^2}$

4.3

19 (cont.)

$$= \frac{2x^3 - 8x^2 + 8x - 3}{(x-2)^2} \quad \begin{matrix} f \\ g \end{matrix}$$

$$y'' = \frac{(6x^2 - 16x + 8)(x-2)^2 - 2(x-2)(2x^3 - 8x^2 + 8x - 1)}{(x-2)^4}$$

2, 1,

CONCAVE UP	CONCAVE DOWN	CONCAVE UP
$(-\infty, 1)$	$(1, 2)$	$(2, \infty)$
TEST POINT $x = 0$	TEST POINT $= 1.5$	TEST POINT $= 3$

1 2

$(1, 1)$

(2,

4.3

13) $y = xe^{x^3}$

$y' = e^x + xe^x$

$y'' = e^x + e^x + xe^x$

$= 2e^x + xe^x = 0$

$e^x(2+x) = 0$

$e^x = 0$ ✗ $\quad 2+x=0$

$x = -2$

CONCAVE DOWN	CONCAVE UP
$f''(x) < 0$	$f''(x) > 0$
-3	0

-2

INFL. PT $\left(-2, \frac{-2}{e^2}\right)$

19) $y = \dfrac{x^3 - 2x^2 + x - 1}{x-2}$ f / g

$y' = \dfrac{(3x^2-4x+1)(x-2) - (x^3-2x^2+x-1)}{(x-2)^2}$

$y' = \dfrac{(3x^3-4x^2+x-6x^2+8x-2) - (x^3-2x^2+x-1)}{(x-2)^2}$

$y' = \dfrac{2x^3-8x^2+8x-1}{(x-2)^2}$ f / g

$A - B = 0$

$A = B$

$y'' = \dfrac{\overbrace{(6x^2-16x+8)(x-2)^2}^{A} - \overbrace{2(x-2)(2x^3-8x^2+8x-1)}^{B}}{((x-2)^2)^2} = 0$

$x \neq 2$

CONCAVE UP	CONCAVE DOWN	CONCAVE UP
$f''(x) > 0$	$f''(x) < 0$	$f''(x) > 0$
0	1.5	3

1 $\quad$ (2)

INFL. $(1,1)$

4.3

9

$y = 2x^{1/5} + 3$

$y' = \frac{2}{5}x^{-9/5}$

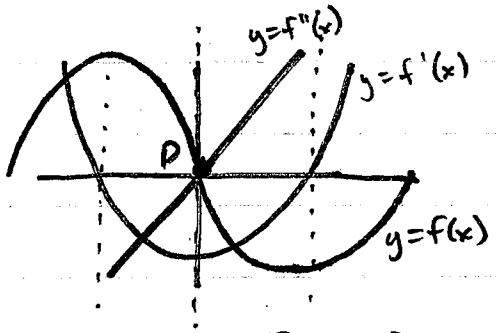
$y'' = \frac{-8}{25}x^{-9/5} = 0$ ← $x \neq 0$

$\frac{-8}{25x^{9/5}}$

CONCAVE UP	CONCAVE DOWN
$f''(x) > 0$	$f''(x) < 0$
-1	1
	0

INFL. PT. $(0, 3)$

41



INCR DECR DECR INCR

DOWN DOWN UP UP

4

$y = xe^{1/x}$ (f & g)

$y' = e^{1/x} + e^{1/x}\left[\frac{-1}{x^2}\right] \cdot x = 0$ $\quad x \neq 0$

$e^{1/x}\left(1 - \frac{1}{x}\right) = 0$ ✓

$1 - \frac{1}{x} = 0$

$1 = \frac{1}{x}$

$x = 1$

INCR	DECR	INCR
$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$
-1	$\frac{1}{2}$	2
	0	1

LOCAL MAX $(0, 0)$

LOCAL MIN $(1, e)$

21 (a) $0 \to -1, 1$, $+ \to (-\infty, -1) \cup (1, \infty)$, $- \to (-1, 1)$

(b) $0 \to 0$, $+ \to (0, \infty)$, $- \to (-\infty, 0)$

4.3

12 $y = e^x$ $[0, 2\pi]$ CONCAVE UP

$y' = e^x$

$y'' = e^x = 0$

14 $y = x\sqrt{9-x^2}$

$= x(9-x^2)^{1/2}$

$y' = (9-x^2)^{1/2} + \frac{1}{2}(9-x^2)^{-1/2}[-2x] \cdot x$

$= (9-x^2)^{1/2} - x^2(9-x^2)^{-1/2}$

$y'' = \frac{1}{2}(9-x^2)^{-1/2}[-2x] + (-2x)(9-x^2)^{-1/2} + -\frac{1}{2}(9-x^2)^{-3/2}[-2x](-x^2)$

$= \frac{-x}{\sqrt{9-x^2}} + \frac{-2x}{\sqrt{9-x^2}} + \frac{-x^3}{\sqrt{(9-x^2)^3}} = 0$ DOMAIN $(-3, 3)$

	CONCAVE UP	CONCAVE DOWN
	$f''(x) > 0$	$f''(x) < 0$

-1 $\quad$ 0 $\quad$ 1

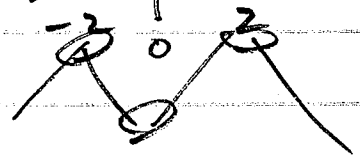
INFL. PT. $(0,0)$

23

INCR	DECR	INCR	DECR
$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$	$f'(x) < 0$
-3	-1	1	3

LOCAL MAX $x = -2, 2$

LOCAL MIN $x = 0$



37 $y = xe^x$

$y' = e^x + xe^x$

$y'' = 2e^x + xe^x$

$y''(-1) = 2e^{-1} + -e^{-1}$

$= e^{-1}$

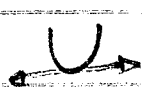
$e^x + xe^x = 0$

$e^x(1+x) = 0$

$e^x = 0$ $\quad$ $1 + x = 0$

$x = -1$

LOCAL MIN $(-1, -\frac{1}{e})$



4.3

51

INCR	DECR	DECR
$f'(x)>0$	$f'(x)<0$	$f'(x)<0$
0	1.5	3

LOCATE MAX (1,2)