4.5

11 $f(x) = x^3 - 2x + 3$, $a = 2$ $\qquad L(x) = 7 + 10(x-2)$

$f(2) = 2^3 - 2(2) + 3 = 7$ $\qquad = 7 + 10x - 20$

$f'(x) = 3x^2 - 2$ $\qquad = 10x - 13$

$f'(2) = 3(2)^2 - 2 = 10$

$f(2.1) = 8.061$ $\qquad L(2.1) = 10(2.1) - 13 = 8$

$|8.161 - 8| = .061$

7 $f(x) = (1+x)^k$, $a = 0$ $\qquad L(x) = 1 + k(x-0)$

$f(0) = (1+0)^k = 1^k = 1$ $\qquad = 1 + kx$

$f'(x) = k(1+x)^{k-1}$

$f'(0) = k(1+0)^{k-1} = k\,1^{k-1} = k$

9 (b) $f(x) = \frac{2}{1-x} = 2(1-x)^{-1}$ $\qquad \boxed{(1+x)^k} \approx 1 + kx$

$= 2(1+(-x))^{-1}$

$= 2(1 + (-1)(-x))$

$= 2(1+x)$

$= 2 + 2x$

13 $\sqrt[3]{998}$ $\quad a = 1000$ $\qquad L(x) = 10 + \frac{1}{300}(998-1000)$

$y = x^{1/3}$ $\qquad = 10 + \frac{1}{300}(-2)$

$y(1000) = 1000^{1/3} = 10$ $\qquad = 10 - \frac{2}{300}$

$y' = \frac{1}{3}x^{-2/3}$ $\qquad = 10 - \frac{1}{150}$

$y'(1000) = \frac{1}{3}(1000)^{-2/3} = \frac{1}{300}$ $\qquad = \frac{1499}{150} \approx 9.993$

1-50, 57-65

4.5

25 $y + xy - x = 0$

$\frac{dy}{dx} + y + x\frac{dy}{dx} - 1 = 0$

$\frac{dy}{dx}(1+x) = 1 - y$

$\frac{dy}{dx} = \frac{1-y}{1+x}$

$= \frac{1 - \left(\frac{x}{1+x}\right)}{1+x}$

$y(1+x) = x$

$y = \frac{x}{1+x}$

$y' = \frac{1(1+x) - 1(x)}{(1+x)^2}$

$y' = \frac{1+x-x}{(1+x)^2}$

$\frac{dy}{dx} = \frac{1}{(1+x)^2}$

$dy = \frac{1}{(1+x)^2}\,dx$

47 $C = 2\pi r$

$\frac{dC}{dr} = 2\pi$

$dC = 2\pi\, dr$

$2 = 2\pi\, dr$

$\frac{1}{\pi} = dr$

$\boxed{\frac{2}{\pi} = dd}$

$A = \pi r^2$

$= \pi\left(\frac{d}{2}\right)^2$

$\frac{dA}{dd} = 2\pi\left(\frac{d}{2}\right)\cdot\frac{1}{2}$

$dA = \left[\pi\left(\frac{d}{2}\right)\right]dd$

$= \cancel{\pi}\left(\frac{10}{\cancel{2}}\right)\cdot\frac{\cancel{2}}{\cancel{\pi}}$

$= 10 \text{ in}^2$

27 $y = (1-x^2)^{1/2}$

$\frac{d}{dx}$

$\frac{dy}{dx} = \cancel{\frac{1}{2}}(1-x^2)^{-1/2}[-\cancel{2}x]$

$dy = \frac{-x}{\sqrt{1-x^2}}\,dx$

33 $f(x) = x^{-1}$, $a = .5$, $dx = .05$

(a) $f(.55) - f(.5) \approx -.181$

(b) $y = x^{-1}$

$\frac{dy}{dx} = -x^{-2}$

$dy = -x^{-2}\,dx$

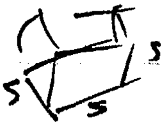
$-(.5)^{-2}(.05) = -.2$

(c) $|-.181 - -.2|$

$.019$

4.5

43) $V = s^3$



$$\frac{dV}{ds} = 3s^2$$

$$dV = 3s^2\,ds$$

$$= 3(15)^2(.2)$$

$$= 135$$

49) $A = s^2$

$$\frac{dA}{ds} = 2s$$

$$dA = 2s\,ds$$

actual value

$$|dA| \leq .02A$$

$$|2s\,ds| = .02s^2$$

$$ds = \frac{.02s^2}{2s}$$

$$ds = .01s$$

4.5

17 $x^2-2x+1=\sin x$

y_1 $x^2-2x+1-\sin x=0$

y_2 $2x-2-\cos x$

1 $f(x)=x^3-2x+3$

$L(x)=f(a)+f'(a)(x-a)$

$f(2)=2^3-2(2)+3=7$

$=7+10(x-2)$

$f'(x)=3x^2-2$

$=7+10x-20$

$f'(2)=3(2)^2-2=10$

$=10x-13$

$L(2.1)=10(2.1)-13=8$

$f(2.1)=(2.1)^3-2(2.1)+3$

$8.061-8=.061/8.061=.8\%$

25 $\overset{f}{y}+x\overset{g}{y}-x=0 \longrightarrow y(1+x)=x$

$\frac{dy}{dx}+y+x\frac{dy}{dx}-1=0$ $\quad y=\frac{x}{1+x}$

$\frac{dy}{dx}(1+x)=1-y$

$\frac{dy}{dx}=\frac{1-y}{1+x}$

$\frac{dy}{dx}=\frac{1-\frac{x}{1+x}}{1+x}$

$dy=\frac{1-\frac{x}{1+x}}{1+x}dx$

$=\frac{\frac{1+x}{1+x}-\frac{x}{1+x}}{1+x}dx$

$=\frac{\frac{1}{1+x}}{1+x}dx$

$=\frac{1}{(1+x)^2}dx \longrightarrow \frac{1}{(1+0)^2}(.01)=.01$

4.5

35 $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$dV = 4\pi r^2 dr$

$4\pi a^2 dr$

$4\pi (10)^2 (.05) = 20\pi \text{ cm}^3$

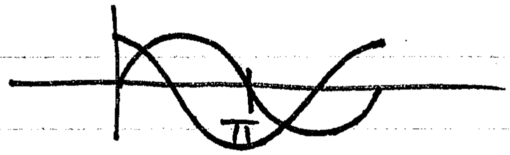
41 $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$dA = 2\pi r \, dr$

$= 2\pi (10)(.1) = 2\pi \text{ in}^2$

43 $y = e^{\sin x}$



$\frac{dy}{dx} = \cos x \, e^{\sin x}$

$dy = \cos x \, e^{\sin x} dx$

$= \cos \pi \, e^{\sin \pi} (-.1)$

$= -1 (1)(-.1) = .1$

27 $y = \sqrt{1-x^2} = (1-x^2)^{1/2}$

$\frac{dy}{dx} = \cancel{\frac{1}{2}}(1-x^2)^{-\frac{1}{2}} [-\cancel{2}x]$

$= \frac{-x}{\sqrt{1-x^2}}$

$dy = \frac{-x}{\sqrt{1-x^2}} dx$

4.5

36 $S = 4\pi r^2$

$\frac{dS}{dr} = 8\pi r$

$dS = 8\pi r\,dr$

$= 8\pi a\,dr$

$= 8\pi(10)(.05) = 4\pi\ cm^2$