

4.6

17) $V = \frac{1}{3} \pi r^2 h$ ✓

~~$V = \frac{1}{3} \pi r^2 h$~~

$V = \frac{1}{3} \pi \left(\frac{45}{6} h\right)^2 h$

$V = \frac{225}{4} \pi h^3$

$V = \frac{225}{4} \pi h^3$

$\frac{dV}{dt} = \frac{225}{4} \pi h^2 \frac{dh}{dt}$

$-50 = \frac{225}{4} \pi (5)^2 \frac{dh}{dt}$

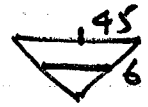
$\frac{dh}{dt} = -0.11 \text{ m/min}$

$= |-1.113 \text{ cm/min}| = 1.113 \text{ cm/min}$



$\frac{h}{r} = \frac{6}{45}$

$\frac{r}{h} = \frac{45}{6}$



$r = \frac{45}{6} h$

$\frac{dr}{dt} = \frac{45}{6} \frac{dh}{dt}$

$\frac{dr}{dt} = \frac{15}{2} (1.113)$

$= -8.488 \frac{\text{cm}}{\text{min}}$

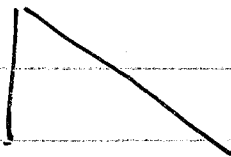
$\frac{45}{6} \cdot 5$

$V = \frac{1}{3} \pi r^2 h$

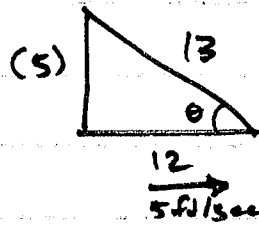
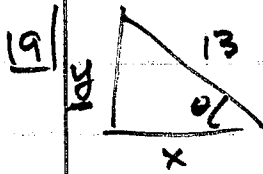
$\frac{dV}{dt} = \frac{1}{3} \pi \left[2r h \frac{dr}{dt} + r^2 \frac{dh}{dt} \right]$

$-50 = \frac{1}{3} \pi \left[2 \left(\frac{75}{2}\right) (5) \frac{dr}{dt} + \left(\frac{75}{2}\right)^2 \frac{dh}{dt} \right]$

19)



4.6



$$(a) \quad x^2 + y^2 = 13^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2(12)(5) + 2(5) \frac{dy}{dt} = 0$$

$$-2(12)(5) \qquad \qquad \qquad -2(5) \frac{dy}{dt}$$

$$\frac{2(5) \frac{dy}{dt}}{2(5)} = \frac{-2(12)(5)}{2(5)}$$

$$= |-12 \text{ ft/sec}| = 12 \text{ ft/sec} = \frac{dy}{dt}$$

(b) $A = \frac{1}{2}xy$

$$\frac{dA}{dt} = \frac{1}{2} \left[y \frac{dx}{dt} + x \frac{dy}{dt} \right]$$

$$= \frac{1}{2} [(5)(5) + (12)(-12)]$$

$$= \boxed{-59.5 \text{ ft}^2/\text{sec}}$$

(c) $\sin \theta = \frac{y}{13}$

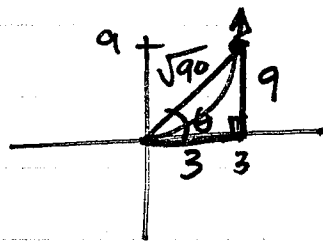
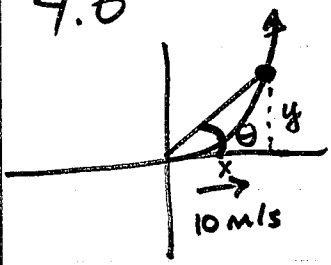
$$\cos \theta \frac{d\theta}{dt} = \frac{1}{13} \frac{dy}{dt}$$

$$\frac{12}{13} \frac{d\theta}{dt} = \frac{1}{13} (-12) \cdot \frac{1}{12}$$

$$\frac{d\theta}{dt} = -1 \text{ radians/sec}$$

25

4.6



$$\text{Sec} = \frac{\text{hyp}}{\text{adj}}$$

$$y = x^2$$

$$\frac{dy}{dt} = 2x \frac{dx}{dt}$$

$$= 2(3)(10)$$

$$\tan \theta = \frac{y}{x}$$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{x \frac{dy}{dt} - y \frac{dx}{dt}}{x^2}$$

$$\left(\frac{\sqrt{90}}{3}\right)^2 \frac{d\theta}{dt} = \frac{3(60) - 9(10)}{3^2}$$

$$\left(\frac{90}{9}\right) \frac{d\theta}{dt} = \frac{90}{9}$$

$$\frac{d\theta}{dt} = 1 \text{ radian/sec}$$

27

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-8 = 4\pi (10)^2 \frac{dr}{dt}$$

$$\frac{-8}{4\pi(100)} = \frac{dr}{dt}$$

$$\frac{-1}{50\pi} = \frac{dr}{dt} \frac{\text{cm}}{\text{min}}$$

$$SA = 4\pi r^2$$

$$\frac{dSA}{dt} = 8\pi r \frac{dr}{dt}$$

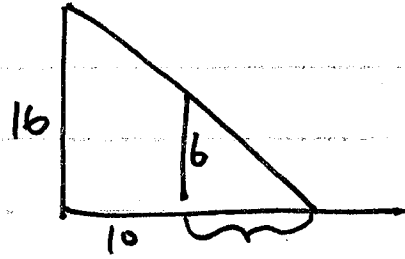
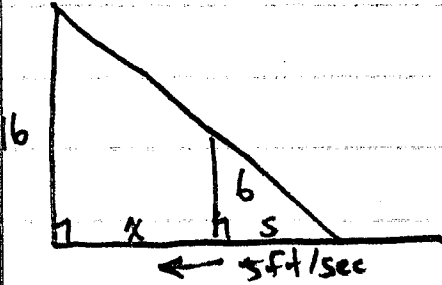
$$= 8\pi(10) \left(\frac{-1}{50\pi}\right)$$

$$= -\frac{8}{5} \text{ cm}^2/\text{min}$$

decrease $\frac{8}{5} \text{ cm}^2/\text{min}$

4.6

29



$$\frac{s}{6} = \frac{s+x}{16}$$

$$\frac{1}{6} s^2 = \frac{1}{16} s^2 + \frac{1}{16} x^2$$

$$\frac{5}{48} s^2 = \frac{1}{16} x^2$$

$$\frac{5}{48} \frac{ds}{dt} = \frac{1}{16} \frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{1}{16} (-8) \cdot \frac{48}{5} = \boxed{-3 \text{ ft/sec}}$$

4.6

(11) $V = \frac{4}{3}\pi r^3$
 (a) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{100\pi}{100\pi} = \frac{4\pi(5)^2}{100\pi} \frac{dr}{dt}$

$1 = \frac{dr}{dt} \quad 1 \text{ ft/min}$

(b) $S = 4\pi r^2$
 $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$= 8\pi(5)(1)$

$= 40\pi \text{ ft}^2/\text{min}$

16 (a) $V = \frac{1}{3}\pi r^2 h$

$h = \frac{2}{3}d$

$V = \frac{1}{3}\pi \left(\frac{4}{3}h\right)^2 h$

$h = \frac{3}{8}(2r)$

$V = \frac{16}{27}\pi h^3$

$h = \frac{3}{4}r \rightarrow h = \frac{3}{4}(4) = 3$

$\frac{dV}{dt} = \frac{16}{9}\pi h^2 \frac{dh}{dt}$

$\frac{4}{3}h = r$

$10 = \frac{16}{9}\pi(3)^2 \frac{dh}{dt}$

9. $10 = \frac{16\pi}{9} \frac{dh}{dt} \cdot 9$

$\frac{90}{16\pi} = \frac{dh}{dt}$

$\frac{90}{256\pi} = \frac{dh}{dt}$

$\frac{90}{256\pi} \text{ m/min} \times 100 = \frac{9000}{256\pi} = \frac{1125}{32\pi} = \frac{dh}{dt}$

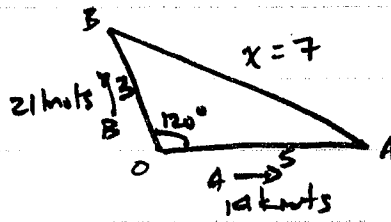
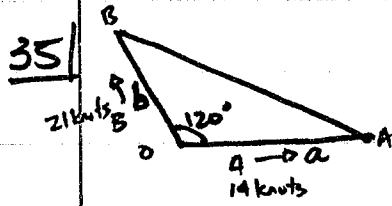
(b) $\frac{4}{3}\pi h^3 = \frac{4}{3}\pi r^3$

$\frac{dh}{dt} = \frac{dr}{dt}$

$\frac{1125}{32\pi} = \frac{375}{8\pi} = \frac{dr}{dt}$

$$x^2 = 3^2 + 5^2 - 2(3)(5)\cos 120^\circ$$

4.6



$$x^2 = a^2 + b^2 - 2ab\cos 120^\circ$$

$$2x \frac{dx}{dt} = 2a \frac{da}{dt} + 2b \frac{db}{dt} - 2\cos 120^\circ (b \frac{da}{dt} + a \frac{db}{dt})$$

$$2(7) \frac{dx}{dt} = 2(5) \frac{da}{dt} + 2(3) \frac{db}{dt} - 2\cos 120^\circ (3(14) + 5(21))$$

$$14 \frac{dx}{dt} = 413$$

$$\frac{dx}{dt} = \frac{413}{14} = \frac{59}{2} \text{ knots}$$

43

$$c(x) = x^3 - 6x^2 + 15x$$

$$\frac{dc}{dt} = 3x^2 \frac{dx}{dt} - 12x \frac{dx}{dt} + 15 \frac{dx}{dt}$$

$$= 3(2)^2 (.1) - 12(2)(.1) + 15(.1)$$

$$\frac{dc}{dt} = .3$$

$$r(x) = 9x$$

$$\frac{dr}{dt} = 9 \frac{dx}{dt}$$

$$= 9(.1)$$

$$= .9$$

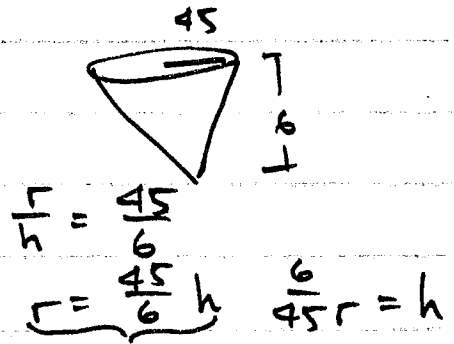
$$D = r - c$$

$$\frac{dD}{dt} = \frac{dr}{dt} - \frac{dc}{dt}$$

$$= .9 - .3 = .6$$

4.6

17) $V = \frac{1}{3} \pi r^2 h$
 (a) $= \frac{1}{3} \pi \left(\frac{45}{6} h\right)^2 h$
 $= \frac{1}{3} \pi \frac{2025}{36} h^2 \cdot h$
 $= \frac{675}{36} \pi h^3$
 $= \frac{75}{4} \pi h^3$



$$\frac{dV}{dt} = \frac{225}{4} \pi h^2 \frac{dh}{dt}$$

$$-50 = \frac{225}{4} \pi (5)^2 \frac{dh}{dt}$$

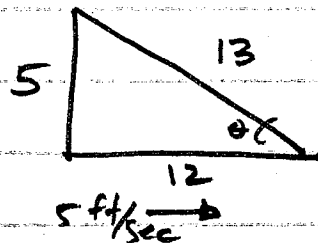
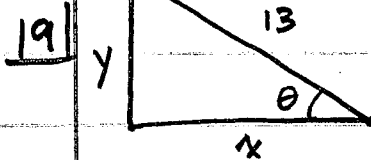
(b) $r = \frac{45}{6} h$
 $\frac{dr}{dt} = \frac{45}{6} \frac{dh}{dt}$
 $= \frac{45}{6} \cdot \frac{32}{9\pi}$

$$-50 = \frac{5625}{4} \pi \frac{dh}{dt} \quad \frac{4}{5625\pi}$$

$$\frac{-8}{225\pi} = \frac{dh}{dt}$$

$$= \frac{-80}{3\pi} \text{ cm/min}$$

$$\frac{-8}{225\pi} \times 100 = \frac{32}{9\pi} \text{ cm/min}$$



(a) $x^2 + y^2 = 13^2$
 $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$
 $2(12)(5) + 2(5) \frac{dy}{dt} = 0$

(b) $A = \frac{1}{2} xy$
 $\frac{dA}{dt} = \frac{1}{2} \left(y \frac{dx}{dt} + x \frac{dy}{dt} \right)$
 $= \frac{1}{2} (5(5) + 12 \frac{dy}{dt})$
 $= \frac{1}{2} (25 - 144)$
 $= -59.5 \text{ ft}^2/\text{sec}$

$$120 + 10 \frac{dy}{dt} = 0$$

$$10 \frac{dy}{dt} = -120$$

$$\frac{dy}{dt} = -12 \text{ ft/sec}$$

$$\boxed{12 \text{ ft/sec}}$$

(c) $\cos \theta = \frac{x}{13}$

$$-\sin \theta \frac{d\theta}{dt} = \frac{1}{13} \frac{dx}{dt}$$

$$\frac{d\theta}{dt} = -1 \text{ radian/sec}$$

$$-\frac{5}{13} \frac{d\theta}{dt} = \frac{1}{13} (5)$$