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$$48) \int_2^5 f(x) dx = 12 \quad \int_5^8 f(x) dx = 4$$

$$(A) \int_2^8 f(x) dx = \int_2^5 f(x) dx + \int_5^8 f(x) dx = 12 + 4 = 16 \checkmark$$

$$(B) \int_2^5 f(x) dx - \int_5^8 3f(x) dx = \int_2^5 f(x) dx - 3 \int_5^8 f(x) dx \\ = 12 - 3(4) = 0 \checkmark$$

$$(C) \int_5^2 f(x) dx = - \int_2^5 f(x) dx = -12 \checkmark$$

$$(D) \int_{-5}^{-8} f(x) dx =$$

$$(E) \int_2^6 f(x) dx + \int_6^8 f(x) dx = \int_2^8 f(x) dx = 16 \checkmark$$

$$27) \int_{-1}^1 \frac{1}{1+x^2} dx = \int_{-1}^1 \tan^{-1} x \quad \tan^{-1}(1) - \tan^{-1}(-1) \\ \frac{\pi}{4} - -\frac{\pi}{4} = \boxed{\frac{\pi}{2}}$$

$$\begin{array}{l} x^2 \\ x^3 \\ \frac{1}{3} x^3 \end{array} \Big|_{-1}^1$$

cos x

-sin x

-cos x

sin x

$$y = \sin x, [0, \pi]$$

$$\frac{1}{\pi - 0} \int_0^{\pi} \sin x dx$$

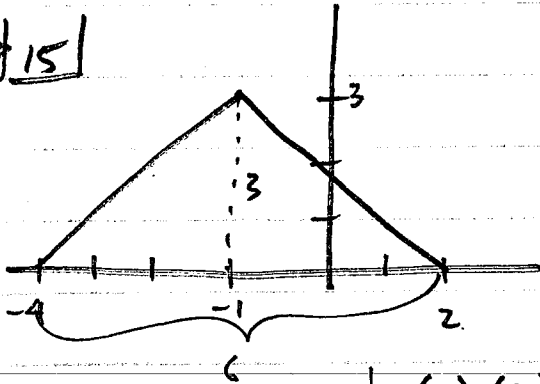
$$\frac{1}{\pi} \Big|_0^{\pi} -\cos x$$

$$\frac{1}{\pi} (-\cos \pi - -\cos(0))$$

$$\frac{1}{\pi} (-(-1) + 1) = \boxed{\frac{2}{\pi}}$$

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$$\frac{1}{2} \int_{-4}^2 f(x) dx$$
$$\frac{1}{6} (9) = \boxed{\frac{3}{2}}$$

$$\frac{1}{2} (6)(3) = 9$$

$$\frac{1}{6} \left[\int_{-4}^{-1} (x+4) dx + \int_{-1}^2 (-x+2) dx \right]$$

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$$\frac{1}{b-a} \int_a^b f(x) dx = f(c)$$

$$\frac{1}{2-(-1)} \int_{-1}^2 (3x^2+2x) dx$$

$$\frac{1}{3} \int_{-1}^2 x^3+x^2$$

$$\frac{1}{3} [(2^3+2^2) - ((-1)^3+(-1)^2)]$$

$$\frac{1}{3} [12-0] = \boxed{4}$$

$$\int_a^b f(x) dx = f(c)(b-a)$$

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3) $\int_1^2 f(x) dx = 5$

a) $\int_1^2 f(u) du = 5$

b) $\int_1^2 \sqrt{3} f(z) dz = 5\sqrt{3}$

c) $\int_2^1 f(t) dt = -5$

d) $\int_1^2 [-f(x)] dx = -5$

4) $\int_{-3}^0 g(t) dt = \sqrt{2}$

a) $\int_0^{-3} g(t) dt = -\sqrt{2}$

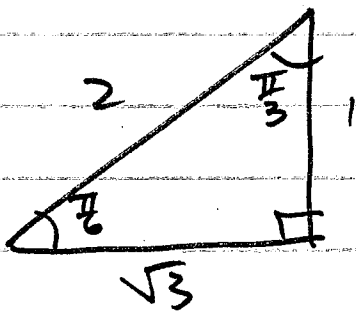
b) $\int_{-3}^0 g(u) du = \sqrt{2}$

c) $\int_{-3}^0 [-g(x)] dx = -\sqrt{2}$

d) $\int_{-3}^0 \frac{g(r)}{\sqrt{2}} dr = \frac{1}{\sqrt{2}} \cdot \sqrt{2} = 1$

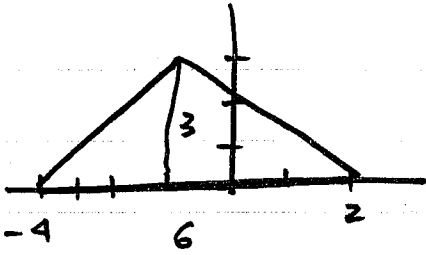
27) $\int_{-1}^1 \frac{1}{1+x^2} dx = \left|_{-1}^1 \tan^{-1} x \right. \quad \tan^{-1} 1 - \tan^{-1}(-1)$
 $\frac{\pi}{4} - -\frac{\pi}{4} = \frac{\pi}{2}$

28) $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}} dx = \left|_0^{1/2} \sin^{-1} x \right. \quad \sin^{-1} \frac{1}{2} - \sin^{-1} 0$
 $\frac{\pi}{6} - 0 = \frac{\pi}{6}$



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$$A = \frac{1}{2}(6)(3) = 9$$

$$f(x) = \begin{cases} x+4, & -4 \leq x \leq -1 \\ -x+2, & -1 \leq x \leq 2 \end{cases}$$

$$\frac{1}{b-a} \int_a^b f(x) dx$$

$$\frac{1}{2-(-4)} \int_{-4}^2 f(x) dx$$

$$\frac{1}{6} \cdot 9 = \frac{3}{2}$$

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TOTAL AMOUNT OF WATER - $1000 + 1000 = 2000 \text{ m}^3$

TOTAL AMOUNT OF TIME

$$\frac{1000}{100} = 100 \text{ MIN}$$

$$\frac{1000}{20} = 50 \text{ MIN}$$

$$150 \text{ MIN}$$

$$\text{AVERAGE RATE} = \frac{\text{TOTAL}}{\text{TIME}} = \frac{2000}{150} = 13.3$$

12

$$y = -\frac{x^2}{2}, [0, 3]$$

$$f(c) = \frac{1}{3-0} \int_0^3 -\frac{x^2}{2} dx$$

$$\frac{1}{3} \left[-\frac{1}{6} (3)^3 - \left(-\frac{1}{6} (0)^3 \right) \right]$$

$$\frac{1}{3} \left[-\frac{9}{2} \right] = -\frac{3}{2}$$

$$-\frac{x^2}{2} = -\frac{3}{2}$$

$$x^2 = 3$$

$$x = \pm \sqrt{3}$$

$$c = \sqrt{3}$$

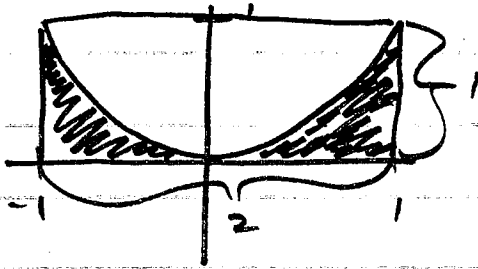
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16) $f(t) = 1 - \sqrt{1-t^2}$, $[-1, 1]$

$$\frac{1}{1-1} \int_{-1}^1 f(x) dx$$

$$\frac{1}{2} \left[2 - \frac{\pi}{2} \right]$$

$$1 - \frac{\pi}{4}$$



$$A = \pi r^2 = \pi(1)^2 = \pi / 2$$

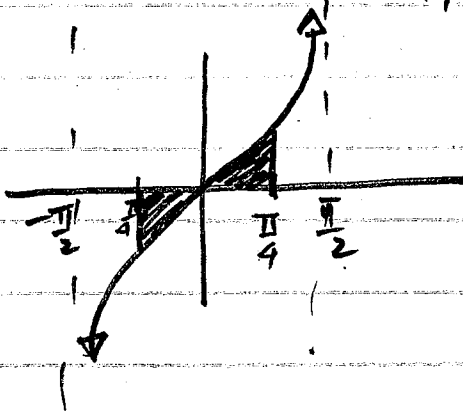
$$\int_{-1}^1 f(x) dx = \text{RECTANGLE} - \text{CIRCLE}$$

$$2 - \frac{\pi}{2}$$

18) $f(\theta) = \tan \theta$, $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

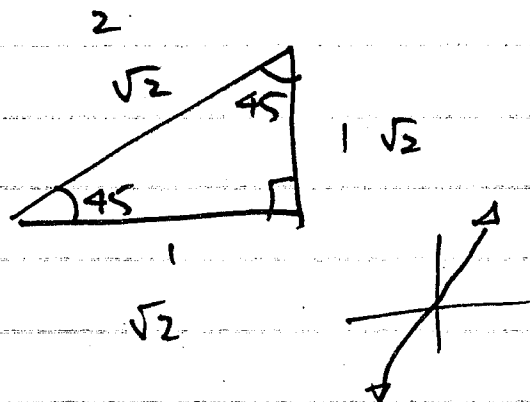
$$\frac{1}{\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan \theta d\theta$$

$$\frac{1}{\frac{\pi}{2}} \cdot 0 = \boxed{0}$$



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22) $\int_{-\pi/4}^{\pi/4} \sec^2 x \, dx$
 $\int_{-\pi/4}^{\pi/4} \tan x$
 $\tan \frac{\pi}{4} - \tan 0$
 $1 - 0 = 1$



27) $\int_{-1}^1 \frac{1}{1+x^2} \, dx$
 $\int_{-1}^1 \tan^{-1} x$
 $\tan^{-1}(1) - \tan^{-1}(-1)$
 $\frac{\pi}{4} - -\frac{\pi}{4} = \frac{\pi}{2}$

$\int \frac{1}{1+x^2} \, dx$
 $\tan^{-1} x + C$