6.2

47 $\int \sin^3 2x\,dx \qquad \sin^2 2x = 1-\cos^2 2x$

$\int \sin 2x \underline{\sin^2 2x}\,dx$

$\int \sin 2x(1-\cos^2 2x)\,dx$

$\frac{1}{2}\int \sin 2x\,dx^{2} - \frac{1}{2}\int \sin 2x\cos^2 2x\,dx^{[2]}$

$du=2dx \leftarrow u=2x \qquad u=2x \quad du=2dx$

$\frac{1}{2}\int \sin u\,du + \frac{1}{2}\int \sin u \cos^2 u\,du\,[-1]\;dv$

$-\frac{1}{2}\cos u \qquad v=\cos u$

$-\frac{1}{2}\cos 2x \qquad dv=-\sin u\,du$

$+\frac{1}{2}\int v^2\,dv$

$+\frac{1}{2}\frac{1}{3}v^3 + C$

$\frac{1}{6}\cos^3 u + C$

$$\boxed{-\frac{1}{2}\cos 2x + \frac{1}{6}\cos^3 2x + C}$$

6.2

37| $\int \frac{\ln^6 x}{x}\,dx$ du

$\ln^6 x = (\ln x)^6$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int u^6\,du$

$\frac{1}{7}u^7 + C$

$\boxed{\frac{1}{7}\ln^7 x + C}$

45| $\int \sec x\,dx$

$\int \sec x\left(\frac{\sec x + \tan x}{\sec x + \tan x}\right)dx$

$\int \frac{\sec^2 x + \sec x \tan x\,dx}{\sec x + \tan x}$ du

$u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x\,dx$

$\int \frac{1}{u}\,du$

$\ln|u| + C$

$\boxed{\ln|\sec x + \tan x| + C}$

39| $\int \frac{dx}{x\ln x}$ du

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \frac{1}{u}\,du$

$\ln|u| + C$

$\boxed{\ln|\ln x| + C}$

41| $\frac{1}{2}\int \frac{2x\,dx}{x^2+1}$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u| + C$

$\frac{1}{2}\ln(x^2+1) + C$

$\ln\sqrt{x^2+1} + C$

6.2

23 $2\int (1-\cos\frac{t}{2})^2 \sin\frac{t}{2}\,dt\,[\frac{1}{2}]$ du

$u = 1-\cos\frac{t}{2}$

$du = \sin\frac{t}{2}\,[\frac{1}{2}]\,dt$

$2\int u^2\,du$

$2\,\frac{1}{3}u^3 + C$

$\frac{2}{3}(1-\cos\frac{t}{2})^3 + C$

27 $\int \sqrt{\tan x}\,\overset{du}{\sec^2 x\,dx}$

$u = \tan x$

$du = \sec^2 x\,dx$

$\int u^{1/2}\,du$

$\frac{2}{3}u^{3/2} + C$ $\qquad (\ln x)^6$

$\boxed{\frac{2}{3}\tan^{\frac{3}{2}} x + C}$

37 $\frac{1}{2}\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,\overset{du}{dt\,[2]}$

$u = 2t+1$

$du = 2dt$ $\qquad dv$

$-\frac{1}{2}\int \frac{\sin(u)}{\cos^2(u)}\,du \longrightarrow \frac{1}{2}\int \frac{\sin u}{\cos u}\cdot\frac{1}{\cos u}\,du \to \frac{1}{2}\int \tan u \sec u$

$\frac{1}{2}\int \tan u \sec u$

$\frac{1}{2}\sec u + C$

$\boxed{\frac{1}{2}\sec(2t+1)+C}$

$v = \cos u$

$dv = -\sin u\,du$

$-\frac{1}{2}\int v^{-2}\,dv$

$+\frac{1}{2}\,v^{-1} + C = \frac{1}{2}\,\frac{1}{\cos u} + C$ $\boxed{\frac{1}{2}\sec(2t+1)+C}$

6.2

29| $\frac{1}{4}\int \tan(4x+2)\, dx\ 4$ $\quad$ du

$u = 4x+2$

$du = 4\,dx$

$\frac{1}{4}\int \tan u\, du$

$-\frac{1}{4}\int \frac{-\sin u}{\cos u}\, du$ $\quad$ dv

$v = \cos u$

$dv = -\sin u\, du$

$-\frac{1}{4}\int \frac{1}{v}\, dv$

$-\frac{1}{4}\ln|v| + C$

$-\frac{1}{4}\ln|\cos u| + C$

$-\frac{1}{4}\ln|\cos(4x+2)| + C$

$\int \tan(4x+2)\, dx$

$-\frac{1}{4}\int \frac{\sin(4x+2)}{\cos(4x+2)}\, dx\ [-4]$

$u = \cos(4x+2)$

$du = -\sin(4x+2)\,[4]\, dx$

$-\frac{1}{4}\int \frac{1}{u}\, du$

$-\frac{1}{4}\ln|u| + C$

$-\frac{1}{4}\ln|\cos(4x+2)| + C$

21| $3\int \frac{dx}{x^2+9}\ [\frac{1}{3}]$ $\quad u = \frac{x}{3} \to x = 3u$

$3\int \frac{dx\ [\frac{1}{3}]}{(3u)^2+9}$ $\quad du = \frac{1}{3}dx$

$3\int \frac{du}{9u^2+9}$

$3\int \frac{du}{9(u^2+1)}$

$\frac{1}{3}\int \frac{du}{u^2+1}$

$\frac{1}{3}\tan^{-1} u + C$

$\boxed{\frac{1}{3}\tan^{-1}\frac{x}{3} + C}$

$\int \frac{1}{x^2+a}\, dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

6.2

51 $\int \tan^4 x\, dx$

$\int \tan^2 x \tan^2 x\, dx$

$\int \tan^2 x (\sec^2 x - 1)\, dx$

$\int \tan^2 x \sec^2 x\, dx - \int \tan^2 x\, dx$

$\int \tan^2 x \underbrace{\sec^2 x\, dx}_{du} - \int (\sec^2 x - 1)\, dx$

$u = \tan x$ $\qquad -\int \sec^2 x \quad -\int 1\, dx$

$du = \sec^2 x\, dx$ $\qquad -\int du$

$\int u^2\, du$ $\qquad -u$

$\frac{1}{3} u^3$

$\boxed{\frac{1}{3}\tan^3 x - \tan x + x + C}$

69 $y = \ln\left|\frac{\cos 3}{\cos x}\right| + 5$ $\qquad \frac{dy}{dx} = \tan x \quad f(3) = 5$

$y(3) = \ln\left|\frac{\cos 3}{\cos 3}\right| + 5 = 5$

$y = \ln|\cos 3| - \ln|\cos x| + 5$

$y' = -\frac{1}{\cos x}[-\sin x] = \frac{\sin x}{\cos x} = \tan x$

57 $\frac{2}{3}\int_0^1 \frac{10\sqrt{\theta}}{(1+\theta^{3/2})^2}\, d\theta \quad \frac{3}{2}$ du

$u = 1 + \theta^{3/2}$

$du = \frac{3}{2}\theta^{1/2}\, d\theta$

$\frac{20}{3}\int_1^2 u^{-2}\, du$

$\frac{20}{3}\Big|_1^2 -u^{-1}$

$\frac{20}{3}[-2^{-1} - -1^{-1}] = \frac{20}{3}[-\frac{1}{2} + 1] = \boxed{\frac{10}{3}}$

6.2

61| $\int_0^7 \frac{dx}{x+2}$

$u = x+2$

$du = dx$

$\int_2^9 \frac{1}{u}\,du$

$\Big|_2^9 \ln|u|$

$\ln 9 - \ln 2$

$\ln \frac{9}{2}$

$\frac{1}{2}[\ln 10 - \ln 2]$

$\frac{1}{2}[\ln \frac{10}{2}]$

$\frac{1}{2}[\ln 5]$

$\ln\sqrt{5}$

$\sin^{-1}\sqrt{x} = y$

83| $\sqrt{x} = \sin y$, $x = \sin^2 y$, $dx = 2\sin y \cos y\, dy$

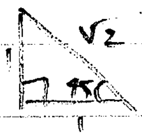
$\sin^{-1}(\frac{1}{\sqrt{2}})$ $\int_0^{1/2} \frac{\sqrt{x}\,dx}{\sqrt{1-x}}$

$\sin^{-1}(0)$

$\int_0^{\frac{\pi}{4}} \frac{\sin y\,(2\sin y \cos y\, dy)}{\sqrt{1-\sin^2 y}}$

$\sin^2 + \cos^2 = 1$

$\cos^2 = 1 - \sin^2$



$\int_0^{\frac{\pi}{4}} \frac{\sin y\,(2\sin y \cos y\, dy)}{\sqrt{\cos^2 y}}$

$\int_0^{\frac{\pi}{4}} \frac{\sin y\,(2\sin y \cancel{\cos y}\, dy)}{\cancel{\cos y}}$

(a) $\int_0^{\frac{\pi}{4}} 2\sin^2 y\, dy$

$\cos 2x = 2\sin^2 x - 1$

$\int_0^{\frac{\pi}{4}} (\cos 2y + 1)\, dy$

$\cos 2x + 1 = 2\sin^2 x$

$\frac{1}{2}\int_0^{\frac{\pi}{4}} \cos 2y\, dy\,[2] + \int_0^{\frac{\pi}{4}} 1\, dy$

$\boxed{\frac{1}{2} + \frac{\pi}{4}}$

$u = 2y$ $\Big|_0^{\frac{\pi}{4}} y = \frac{\pi}{4} - 0$

$du = 2dy$

$\frac{1}{2}[1 - 0] = \frac{1}{2}$

$\frac{1}{2}\int_0^{\frac{\pi}{2}} \cos u\, du = \frac{1}{2}\Big|_0^{\frac{\pi}{2}} \sin u = \frac{1}{2}[\sin\frac{\pi}{2} - \sin 0]$

6.2

37 $\frac{1}{2}\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\, dt\,[2]$ du

$u = 2t+1$

$du = 2\,dt$

$-\frac{1}{2}\int \frac{\sin u}{\cos^2 u}\, du\,[-1]$ du

$v = \cos u$

$dv = -\sin u\, du$

$-\frac{1}{2}\int \frac{1}{v^2}\, dv$

$-\frac{1}{2}\int v^{-2}\, dv$

$\frac{1}{2}\; v^{-1} + C$

$\frac{1}{2}\,\frac{1}{\cos u} + C$

$\frac{1}{2\cos(2t+1)} + C = \frac{1}{2}\sec(2t+1) + C$

42 $\int \frac{40\, dx}{x^2+25}$ du $u = \frac{x}{5} \to u = \frac{1}{5}x$

$5 \cdot 40 \int \frac{1}{x^2+25}\, dx\, \frac{1}{5}$ $5u = x$ $du = \frac{1}{5}dx$

$200 \int \frac{1}{25u^2+25}$

$200 \int \frac{1}{25(u^2+1)}\, du$

$\frac{200}{25}\int \frac{1}{u^2+1}\, du$

$8 \arctan|u| + C$

$\boxed{8 \arctan \frac{x}{5} + C}$

$\boxed{8 \tan^{-1}\frac{x}{5} + C}$

6.2

33 $\int \frac{\ln^6 x}{x}\,dx = \int \underbrace{\frac{1}{x}(\ln x)^6\,dx}_{du}$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int u^6\,du$

$\frac{1}{7}u^7 + C$

$\frac{1}{7}\ln^7 x + C$

17 $\frac{1}{3}\int \sin 3x\,dx\,[3]$

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}\int \sin u\,du$

$-\frac{1}{3}\cos u + C$

$-\frac{1}{3}\cos 3x + C$

23 $2\int \left(1-\cos\frac{t}{2}\right)^2 \underbrace{\sin\frac{t}{2}\,dt\left[\frac{1}{2}\right]}_{du}$

$u = 1-\cos\frac{t}{2}$

$du = \sin\frac{t}{2}\left[\frac{1}{2}\right]dt$

$2\int u^2\,du$

$2\cdot\frac{1}{3}\cdot u^3 + C$

$\frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + C$

6.2

47| $\int \sin^3 2x \, dx$

$\int \sin 2x \sin^2 2x \, dx$

$\int \sin 2x (1-\cos^2 2x) \, dx$

$\frac{1}{2}\int \sin 2x \, dx^{[2]} - \frac{1}{2}\int \cos^2 2x \sin 2x \, dx^{[2]}$

$u = 2x$

$du = 2dx$

$\frac{1}{2}\int \sin u \, du + \frac{1}{2}\int \cos^2 u \, \sin u \, du$ dv

$-\frac{1}{2}\cos u$

$-\frac{1}{2}\cos 2x$

$u = 2x$

$du = 2dx$

$v = \cos u$

$dv = -\sin u \, du$

$+\frac{1}{2}\int v^2 \, dv$

$+\frac{1}{2} \frac{1}{3} v^3 + C$

$\frac{1}{6}\cos^3 u$

$u = \cos 2x$

$du = -\sin 2x [2] dx$

$+\frac{1}{2}\int u^2 \, du$

$+\frac{1}{2} \frac{1}{3} u^3 + C$

$\frac{1}{6}\cos^3 2x$

$-\frac{1}{2}\cos 2x + \frac{1}{6}\cos^3 2x + C$

29| $\int \tan(4x+2) \, dx$

$\frac{1}{4}\int \frac{\sin(4x+2)}{\cos(4x+2)} \, dx \, [4]$

$u = 4x+2$

$du = 4 \, dx$

$-\frac{1}{4}\int \frac{\sin u}{\cos u} \, du$ dv

$v = \cos u$

$dv = -\sin u \, du$

$-\frac{1}{4}\int \frac{1}{v} \, dv$

$-\frac{1}{4}\ln|v| + C$

$-\frac{1}{4}\ln|\cos u| + C \rightarrow -\frac{1}{4}\ln|\cos(4x+2)| + C$

6.2

39| $\int \frac{dx}{x \ln x} = \int \frac{1}{\ln x} \cdot \underbrace{\frac{1}{x} dx}_{du}$

$u = \ln x$

$du = \frac{1}{x} dx$

$\int \frac{1}{u} du$

$\ln|u| + C$

$\ln|\ln x| + C$

45| $\int \sec x \, dx \left(\frac{\sec x + \tan x}{\sec x + \tan x} \right)$

$\int \frac{\sec^2 x + \sec x \tan x \, dx}{\sec x + \tan x}$ du

$u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x$

$\int \frac{1}{u} du$

$\ln|u| + C$

$\ln|\sec x + \tan x|$

21| $3\int \frac{dx \, \frac{1}{3}}{x^2 + 9}$ $u = \frac{x}{3}$ $du = \frac{1}{3} dx$

$3\int \frac{du}{(3u)^2 + 9}$ $3u = x$

$3\int \frac{du}{9u^2 + 9}$ d

$3\int \frac{du}{9(u^2+1)}$

$\frac{1}{3} \int \frac{du}{u^2+1}$

$\frac{1}{3} \arctan u + C$

$\frac{1}{3} \arctan \frac{x}{3} + C$

$\int \frac{1}{x^2 + a^2} dx$

$\boxed{\frac{1}{a} \tan^{-1} \frac{x}{a} + C}$

6.2

81] $\int \frac{dx}{\sqrt{1-x^2}}$ $\qquad x = \sin u \longrightarrow \sin^{-1} x = u$

$\int \frac{\cos u \, du}{\sqrt{1-\sin^2 u}}$ $\qquad dx = \cos u \, du$

$\int \frac{\cos u \, du}{\sqrt{\cos^2 u}}$

$\int \frac{\cancel{\cos u} \, du}{\cancel{\cos u}}$

$\int 1 \, du$

$u + C$

$\sin^{-1} x + C$

53] $\int_0^3 \sqrt{y+1} \, dy$

$u = y+1$

$du = dy$

$\int_1^4 u^{1/2} \, du$

$\left.\frac{2}{3} u^{3/2}\right|_1^4$

$\frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$

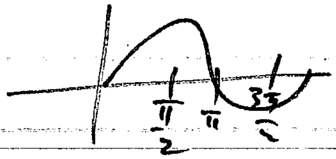
$\frac{16}{3} - \frac{2}{3} = \boxed{\frac{14}{3}}$

6.2

7| $\int \csc^2 u \, du = -\cot u + C$ $\qquad \frac{d}{du}(-\cot u + C)$

$\csc^2 u$

CHAPTER REVIEW



5| $\int_0^{\pi/2} 5 \sin^{3/2} x \, \overbrace{(\cos x \, dx)}^{du}$

$u = \sin x$

$du = \cos x \, dx$ $\qquad \frac{5}{\frac{5}{2}} \quad \cancel{5} \cdot \frac{2}{\cancel{5}} = 2$

$\int_0^1 5u^{3/2} \, du$

$\Big|_0^1 \; 2u^{5/2}$

$2(1)^{5/2} - 2(0)^{5/2} = \boxed{2}$

7| $\int_0^{\pi/4} e^{\tan x} \overbrace{(\sec^2 x \, dx)}^{du}$

$u = \tan x$

$du = \sec^2 x \, dx$

$\int_0^1 e^u \, du$

$\Big|_0^1 \; e^u \qquad e^1 - e^0$

$\boxed{e-1}$