6.3

31| $\frac{dy}{d\theta} = \theta \sec^{-1}\theta \quad , \theta > 1 \qquad u = \sec^{-1}\theta \qquad v = \frac{1}{2}\theta^2$

$\int dy = \int \theta \sec^{-1}\theta \, d\theta \qquad du = \frac{1}{|\theta|\sqrt{\theta^2-1}} d\theta \qquad dv = \theta d\theta$

$y = \frac{1}{2}\theta^2 \sec^{-1}\theta - \int \frac{1}{2}\theta^2 \frac{1}{|\theta|\sqrt{\theta^2-1}} d\theta \qquad du$

$- \frac{1}{2}\cdot\frac{1}{2}\int (\theta^2-1)^{-1/2} \theta \, d\theta \, [2] \qquad u = \theta^2 - 1$

$- \frac{1}{4}\int u^{-1/2} du \qquad du = 2\theta \, d\theta$

$- \frac{1}{4} \cdot 2 u^{1/2} + C$

$\frac{1}{2}\theta^2 \sec^{-1}\theta - \frac{1}{2}\sqrt{\theta^2 - 1} + C$

17| $\int e^x \sin x \, dx$

$u = \sin x \qquad v = e^x$

$du = \cos x \, dx \quad dv = e^x dx$

$\int e^x \sin x = e^x \sin x - \int e^x \cos x \, dx$

$u = \cos x \qquad v = e^x$

$du = -\sin x \qquad dv = e^x dx$

$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x + -\int e^x \sin x \, \cancel{dx}$

$+\int e^x \sin x \, dx \qquad + \int \cancel{e^x} \sin x \, dx$

$\frac{2\int e^x \sin x \, dx}{2} = \boxed{\frac{e^x \sin x - e^x \cos x}{2} + C}$

$\int e^x \sin x = e^x \sin x - [e^x \cos x - \int -e^x \sin x \, dx]$

6.3

25 $\int_0^{\pi/2} x^2 \sin 2x \, dx$

SIGN	u	dv
+	x^2	$\sin 2x$
−	$2x$	$-\frac{1}{2}\cos 2x$
+	2	$-\frac{1}{4}\sin 2x$
−	0	$\frac{1}{8}\cos 2x$

$\Big|_0^{\pi/2} -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x$

$+\frac{1}{4}\cos 2x$

$\left[-\frac{1}{2}\left(\frac{\pi}{2}\right)^2 \overset{-1}{\cos\left(2\cdot\frac{\pi}{2}\right)} + \cancel{\frac{1}{2}\left(\frac{\pi}{2}\right)\sin\left(2\cdot\frac{\pi}{2}\right)} + \frac{1}{4}\overset{-1}{\cos\left(2\cdot\frac{\pi}{2}\right)}\right] -$

$\left[\cancel{-\frac{1}{2}(0)^2\cos(0)} + \cancel{\frac{1}{2}(0)\sin(0)} + \frac{1}{4}\cos(0)\right]$

$\frac{\pi^2}{8} - \frac{1}{4} \qquad -\frac{1}{4}$

$\frac{\pi^2}{8} - \frac{1}{2}$

$\frac{1}{2}\sin 2x\,[2] \quad u = 2x$

$du = 2dx$

33 $y = x\sin x$

(a) $\int_0^{\pi} x\sin x \, dx$

$u = x \qquad v = -\cos x$

$du = dx \qquad dv = \sin x \, dx$

$\Big|_0^{\pi} -x\cos x - \int -\cos x \, dx$

$\Big|_0^{\pi} -x\cos x + \sin x$

$[-\pi\cos\pi + \sin\pi] - [\cancel{-0\cos 0 + \sin 0}] = \pi$

6.3

(b) $\int_{\pi}^{2\pi} x\sin x\,dx$

$\Big|_{\pi}^{2\pi} -x\cos x + \sin x$

$\left[-2\pi\cos 2\pi + \cancel{\sin 2\pi}\right] - \left[-\pi\cos\pi + \cancel{\sin\pi}\right]$

$-2\pi - \pi = -3\pi$ $\quad$ 3π

(c) $\int_{0}^{2\pi} x\sin x\,dx = \int_{0}^{\pi} x\sin x\,dx + \int_{\pi}^{2\pi} x\sin x\,dx$

$\pi + 3\pi = 4\pi$

35) $\frac{1}{b-a}\int_a^b f(x)\,dx$

$\frac{1}{2\pi-0}\int_0^{2\pi} 2e^{-t}\cos t\,dt$

$\frac{1}{2\pi}\cdot 2\int_0^{2\pi} e^{-t}\cos t\,dt$

$u=\cos t \quad v=-e^{-t}$

$du=-\sin t\,dt \quad dv=e^{-t}dt$

$\frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t = -\cos t e^{-t} - \int_0^{2\pi} e^{-t}\sin t\,dt$

$u=\sin t \quad v=-e^{-t}$

$du=\cos t\,dt \quad dv=e^{-t}dt$

$\frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t = \Big|_0^{2\pi} -e^{-t}\cos t - \left[-e^{-t}\sin t - \int_0^{2\pi} -e^{-t}\cos t\,dt\right]$

$\frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t = \Big|_0^{2\pi} -e^{-t}\cos t + e^{-t}\sin t - \cancel{\int_0^{2\pi} e^{-t}\cos t\,dt}$

$+1\int_0^{2\pi} e^{-t}\cos t \qquad + \int_0^{2\pi} e^{-t}\cos t\,dt$

$\frac{\left(\frac{1}{\pi}+1\right)\int_0^{2\pi} e^{-t}\cos t}{\left(\frac{1}{\pi}+1\right)} = \left[-e^{-2\pi}\cos 2\pi + \cancel{e^{-2\pi}\sin 2\pi}\right] - \left[-e^{-0}\cos 0 + \cancel{e^{-0}\sin 0}\right]$

$= \boxed{\frac{-e^{-2\pi}+1}{\left(\frac{1}{\pi}+1\right)}}$

6.3

13| $\frac{du}{dx} = x\sec^2 x \qquad u=1 \text{ when } x=0$

$\int du = \int x\sec^2 x\,dx \qquad u = x \qquad v = \tan x$

$u = x\tan x - \int \tan x\,dx \qquad du = dx \qquad dv = \sec^2 x\,dx$

$+\int \frac{\sin x}{\cos x}\,dx\,[-1]\,du \qquad u = \cos x$

$+\int \frac{1}{u}\,du \qquad du = -\sin x\,dx$

$+\ln|u| + C$

$u = x\tan x + \ln|\cos x| + C$

$1 = \cancel{0\tan 0} + \cancel{\ln|\cos 0|} + C$

$1 = C$

$\boxed{u = x\tan x + \ln|\cos x| + 1}$

29| $\frac{dy}{dx} = x^2 e^{4x}$

$\int dy = \int x^2 e^{4x}\,dx$

$\boxed{y = \frac{1}{4}x^2 e^{4x} - \frac{1}{8}x e^{4x} + \frac{1}{32}e^{4x} + C}$

SIGN	u	dv
+	x^2	e^{4x}
	$2x$	$\frac{1}{4}e^{4x}$
+	2	$\frac{1}{16}e^{4x}$
−	0	$\frac{1}{64}e^{4x}$

55| $\int \cos^{-1} x\,dx$

$u = \cos^{-1} x \qquad v = x$

$du = \frac{-1}{\sqrt{1-x^2}}\,dx \qquad dv = dx$

$x\cos^{-1}x - \frac{1}{2}\int \frac{-x}{\sqrt{1-x^2}}\,dx\,[2]\,du \qquad u = 1-x^2$

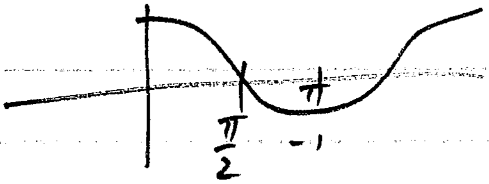
$x\cos^{-1}x - \frac{1}{2}\int u^{-1/2}\,du \qquad du = -2x\,dx$

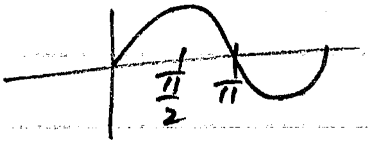
$x\cos^{-1}x - \frac{1}{2}\cdot 2u^{1/2} + C$

$x\cos^{-1}x - \sqrt{1-x^2} + C$

6.3

25| $\int_0^{\pi/2} x^2 \sin 2x \, dx$





SIGN	u	dv
+	x^2	$\sin 2x$
−	$2x$	$-\frac{1}{2}\cos 2x$
+	2	$-\frac{1}{4}\sin 2x$
−	0	$\frac{1}{8}\cos 2x$

$$\Big|_0^{\pi/2} -\frac{1}{2}x^2\cos 2x + \frac{1}{4}(2x)\sin 2x + \frac{1}{4}\cos 2x$$

$$\left[-\frac{1}{2}\left(\frac{\pi}{2}\right)^2 \overbrace{\cos 2\left(\frac{\pi}{2}\right)}^{-1} + \cancel{\frac{1}{2}\left(\frac{\pi}{2}\right)\sin 2\left(\frac{\pi}{2}\right)} + \frac{1}{4}\overbrace{\cos 2\left(\frac{\pi}{2}\right)}^{-1}\right] -$$
$$\left[\cancel{-\frac{1}{2}(0)^2\cos 2(0)} + \cancel{\frac{1}{2}(0)\sin 2(0)} + \frac{1}{4}\cos 2(0)\right]$$

$$\frac{\pi^2}{8} - \frac{1}{4} \qquad -\frac{1}{4}$$

$$\frac{\pi^2}{8} - \frac{1}{2}$$

27| $\int_{-2}^{3} e^{2x}\cos 3x \, dx$

6.3

27) $\int_{-2}^{3} e^{2x}\cos 3x\,dx$

$u = e^{2x}$ $\quad v = \frac{1}{3}\sin 3x$

$du = 2e^{2x}dx$ $\quad dv = \cos 3x\,dx$

$\Big|_{-2}^{3} \frac{1}{3}e^{2x}\sin 3x - \int_{-2}^{3}\frac{2}{3}e^{2x}\sin 3x\,dx$

$u = \frac{2}{3}e^{2x}$ $\quad v = -\frac{1}{3}\cos 3x$

$du = \frac{4}{3}e^{2x}dx$ $\quad dv = \sin 3x\,dx$

$\Big|_{-2}^{3}\frac{1}{3}e^{2x}\sin 3x - \left[-\frac{2}{9}e^{2x}\cos 3x - \int_{-2}^{3}-\frac{4}{9}e^{2x}\cos 3x\,dx\right]$

$\int_{-2}^{3} e^{2x}\cos 3x\,dx = \Big|_{-2}^{3}\frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \cancel{\int_{-2}^{3}\frac{4}{9}e^{2x}\cos 3x\,dx}$

$+\int \frac{4}{9}e^{2x}\cos 3x\,dx \qquad + \cancel{\int\frac{4}{9}e^{2x}\cos 3x\,dx}$

$\frac{9}{13}\cdot\frac{13}{9}\int e^{2x}\cos 3x\,dx = \frac{9}{13}\Big|_{-2}^{3}\frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x$

$\frac{9}{13}\left[\left[\frac{1}{3}e^{2(3)}\sin 3(3) + \frac{2}{9}e^{2(3)}\cos 3(3)\right] - \left[\frac{1}{3}e^{2(-2)}\sin 3(-2) + \frac{2}{9}e^{2(-2)}\cos 3(-2)\right]\right]$

$\frac{9}{13}\left[\left[\frac{1}{3}e^{6}\sin 9 + \frac{2}{9}e^{6}\cos 9\right] - \left[\frac{1}{3}e^{-4}\sin(-6) + \frac{2}{9}e^{-4}\cos(-6)\right]\right]$

6.3

$\int u\,dv = uv - \int v\,du$

17) $\int e^x \sin x\,dx$

$u = e^x \qquad v = -\cos x$

$du = e^x\,dx \qquad dv = \sin x\,dx$

$\int e^x \sin x\,dx = -e^x\cos x + \int + e^x \cos x\,dx$

$u = e^x \qquad v = \sin x$

$du = e^x\,dx \qquad dv = \cos x\,dx$

$\int e^x \sin x\,dx = -e^x\cos x + e^x \sin x - \cancel{\int e^x \sin x\,dx}$

$+ \int e^x \sin x\,dx \qquad + \cancel{\int e^x \sin x\,dx}$

$\frac{2\int e^x \sin x\,dx}{2} = \boxed{\frac{-e^x\cos x + e^x \sin x}{2} + C}$

19) $\int e^x \cos 2x\,dx$

$u = e^x \qquad v = \frac{1}{2}\sin 2x$

$du = e^x\,dx \qquad dv = \cos 2x\,dx$

$\int e^x \cos 2x\,dx = \frac{1}{2}e^x \sin 2x - \int \frac{1}{2}e^x \sin 2x\,dx$

$u = \frac{1}{2}e^x \qquad v = -\frac{1}{2}\cos 2x$

$du = \frac{1}{2}e^x\,dx \qquad dv = \sin 2x\,dx$

$\int e^x \cos 2x\,dx = \frac{1}{2}e^x \sin 2x + \frac{1}{4}e^x \cos 2x + \cancel{\int -\frac{1}{4}e^x \cos 2x\,dx}$

$+\int \frac{1}{4}e^x \cos 2x\,dx \qquad + \cancel{\int \frac{1}{4}e^x \cos 2x\,dx}$

$\cancel{\frac{4}{5}}\,\cancel{\frac{5}{4}}\int e^x \cos 2x\,dx = \boxed{\frac{4}{5}\left[\frac{1}{2}e^x \sin 2x + \frac{1}{4}e^x \cos 2x\right] + C}$

6.3

56 $\int \log_2 x \, dx$

$u = \log_2 x \qquad v = x$

$du = \frac{1}{(\ln 2) x} dx \qquad dv = dx$

$x \log_2 x - \int \cancel{x} \cdot \frac{1}{(\ln 2)\cancel{x}} dx$

$\boxed{x \log_2 x - \frac{1}{\ln 2} x + C}$

11 $\frac{dy}{dx} = (x+2) \sin x \qquad y = 2 \text{ when } x = 0$

$\int dy = \int (x+2) \sin x \, dx$

$y = \qquad u = x+2 \qquad v = -\cos x$

$du = dx \qquad dv = \sin x \, dx$

$y = -(x+2)\cos x - \int -\cos x \, dx$

$y = -(x+2)\cos x + \sin x + C \rightarrow \boxed{y = -(x+2)\cos x + \sin x + 4}$

$2 = -(0+2)\cos(0) + \sin 0 + C$

$2 = -2 + C$

$4 = C$

6.3

24 $\int x^3 \cos 2x \, dx$

SIGN	u	dv
$+$	x^3	$\cos 2x$
$-$	$3x^2$	$\frac{1}{2}\sin 2x$
$+$	$6x$	$-\frac{1}{4}\cos 2x$
$-$	6	$-\frac{1}{8}\sin 2x$
$+$	0	$\frac{1}{16}\cos 2x$

$$\boxed{\frac{1}{2}x^3 \sin 2x + \frac{3}{4}x^2 \cos 2x - \frac{3}{4}x \sin 2x - \frac{3}{8}\cos 2x + C}$$

20 $\int e^{-x} \sin 2x \, dx$ $\qquad uv - \int v \, du$

$u = \sin 2x \qquad v = -e^{-x}$

$du = 2\cos 2x \, dx \quad dv = e^{-x} \, dx$

$$-e^{-x} \sin 2x + \int 2e^{-x} \cos 2x \, dx$$

$u = \cos 2x \qquad v = -2e^{-x}$

$du = -2\sin 2x \, dx \quad dv = 2e^{-x} \, dx$

$$-e^{-x} \sin 2x + -2e^{-x} \cos 2x - \cancel{\int 4e^{-x} \sin 2x \, dx} = \int e^{-x} \sin 2x \, dx$$

$$\cancel{+4\int e^{-x} \sin 2x \, dx} \qquad +4\int e^{-x} \sin 2x \, dx$$

$$\frac{-e^{-x} \sin 2x - 2e^{-x} \cos 2x}{5} = \frac{5\int e^{-x} \sin 2x \, dx}{5}$$

$$\boxed{-\frac{1}{5}e^{-x} \sin 2x - \frac{2}{5}e^{-x} \cos 2x + C}$$