6.4

29 $y = Ce^{kt}$ $\qquad \underline{t = \frac{-3}{k}}$

$\cancel{y = Ce^{k(\frac{-3}{k})}}$

$y = Ce^{k(\frac{-3}{k})}$

$y = Ce^{-3}$

$x^2 \cdot x^5 = x^7$

$x^6 x^1$

$x^5 \cdot x^2$

$x^4 \cdot x^3$

7 $\frac{dy}{dx} = (\cos x) e^{y + \sin x}$ $\qquad \underline{y = 0 \text{ when } x = 0}$

$\frac{dy}{dx} = \frac{(\cos x) e^y e^{\sin x}}{e^y}$

e^y

$\int e^{-y}\, dy = \int (\cos x) e^{\sin x} dx$ $\quad du$

$u = \sin x$

$du = \cos x\, dx \qquad e^u du$

$-e^{-y} = e^{\sin x} + C$

$-e^{-0} = e^{\sin 0} + C$

$-1 = 1 + C$

$-1 \quad -1$

$-2 = C$

$-e^{-y} = e^{\sin x} - 2$

$e^{-y} = -e^{\sin x} + 2$

$-y = \ln|-e^{\sin x} + 2|$

$y = -\ln|-e^{\sin x} + 2|$

$-e^{-y} = e^{\sin x} + C$

$e^{-y} = -e^{\sin x} + C$

$-y = \ln|-e^{\sin x} + C|$

$y = -\ln|-e^{\sin x} + C|$

$0 = -\ln|-1 + C|$

$0 = \ln|-1 + C|$

$1 = -1 + C$

$2 = C$

6.4

21 $\frac{dy}{dt} = -0.0077y$

$y = Ce^{kt}$

$y = Ce^{-0.0077t}$ $\rightarrow$ $\frac{y}{C} = \frac{\cancel{C}e^{-0.0077t}}{\cancel{C}}$

$\frac{y}{c} = \frac{1}{2} = e^{-0.0077t}$ now/started $\frac{y}{C} = e^{-0.0077t}$

$\ln\frac{1}{2} = \ln e^{-0.0077t}$ $\frac{1}{2} = e^{-0.0077t}$

$\frac{\ln\frac{1}{2}}{-0.0077} = \frac{-0.0077t}{-0.0077}$

$90.019 \text{ years} = t$

31 $\ln|y - y_e| = kt + C$

temp @ time t, temp environment, k constant, time, C constant

$\ln|90-20| = k(0) + C$

$\ln 70 = C$

$\ln|60-20| = k(10) + \ln 70$

$\ln 40 = 10k + \ln 70$

$\ln\frac{4}{7} = 10k$

$\frac{\ln\frac{4}{7}}{10} = k$

$\ln|35-20| = \frac{\ln\frac{4}{7}}{10}t + \ln 70$

$\ln 15 = \frac{\ln\frac{4}{7}}{10}t + \ln 70$

$t = \frac{10}{\ln\frac{4}{7}} \cdot \ln\frac{3}{14} = \frac{\ln\frac{4}{7}}{10}t \quad \frac{10}{\ln\frac{4}{7}}$

$\frac{10\ln\frac{3}{14}}{\ln\frac{4}{7}} \approx 27.527 - 10 = \boxed{17.527 \text{ min}}$

$\ln|35 - -15| = \frac{\ln\frac{4}{7}}{10}t + \ln 105$

$\ln|50| = \frac{\ln\frac{4}{7}}{10}t + \ln 105$

$\frac{10}{\ln\frac{4}{7}} \ln\frac{10}{21} = \frac{\ln\frac{4}{7}}{10}t \quad \frac{10}{\ln\frac{4}{7}}$

$\frac{10\ln\frac{10}{21}}{\ln\frac{4}{7}} \approx 13.258 \text{ min}$

6.4

$$k = \frac{\ln \frac{4}{7}}{10}$$

$$\ln|90 - -15| = \frac{\ln \frac{4}{7}}{10}(0) + C$$

$$\ln 105 = C$$

$$\ln|35 - -15| = \frac{\ln \frac{4}{7}}{10} t + \ln 105$$

19) $A = P\left(1 + \frac{r}{n}\right)^{nt}$ $\frac{A}{P} = 2$

(a) $2 = \left(1 + \frac{.0475}{1}\right)^{1t}$

$$2 = (1.0475)^t$$

$$\ln 2 = \ln (1.0475)^t$$

$$\frac{\ln 2}{\ln(1.0475)} = t \approx 14.936 \text{ years}$$

(b) $2 = \left(1 + \frac{.0475}{12}\right)^{12t}$

$$2 = (1.0039)^{12t}$$

$$\ln 2 = \ln (1.0039)^{12t}$$

$$14.621 \approx \frac{\frac{\ln 2}{\ln(1.0039)}}{12} = \frac{12t}{12}$$

9) $\frac{\frac{dy}{dx}}{y^2} = \frac{-2xy^2}{y^2}$ $y = 0.25,\ x = 1$

$$y^{-2}dy = -2x\,dx$$

$$-y^{-1} = -x^2 + C$$

$$-(.25)^{-1} = -(1)^2 + C$$

$$-4 = -1 + C$$

$$+1 \quad\quad +1$$

$$-3 = C$$

$$-y^{-1} = -x^2 - 3$$

$$y^{-1} = x^2 + 3$$

$$y = \frac{1}{x^2 + 3}$$

6.4

35 5700 years = half-life Carbon-14

$y = Ce^{kt}$

$\frac{1}{2} = e^{k(5700)}$

$\ln \frac{1}{2} = \ln e^{k(5700)}$

$\frac{\ln \frac{1}{2}}{5700} = k$

$.445 = e^{\frac{\ln \frac{1}{2}}{5700} t}$

$\ln .445 = \ln e^{\frac{\ln \frac{1}{2}}{5700} t}$

$\frac{5700}{\ln \frac{1}{2}} \ln .445 = \frac{\ln \frac{1}{2}}{5700} t \frac{5700}{\ln \frac{1}{2}}$

$\frac{5700 \ln .445}{\ln \frac{1}{2}} \approx 6658.300$

6.9

37 $y = Ce^{kt}$

$\frac{y}{c} = e^{kt} = \frac{1}{3}$

$e^{k(5)} = \frac{1}{3}$

$\frac{5k}{5} = \frac{\ln\frac{1}{3}}{5}$

$k = \frac{\ln\frac{1}{3}}{5}$

$\frac{y}{c} = e^{kt} = \frac{1}{2}$

$e^{\frac{\ln\frac{1}{3}}{5}t} = \frac{1}{2}$

$\frac{\ln\frac{1}{3}}{5} t = \ln\frac{1}{2}$

$t = \frac{5\ln\frac{1}{2}}{\ln\frac{1}{3}}$

$x^2 \cdot x^3 = x^5$

$x^5 = x^2 x^3$

$= x^1 x^4$

$= x^0 x^5$

6.4

7) $\frac{dy}{dx} = (\cos x) e^{y + \sin x}$

$dx \frac{\frac{dy}{dx}}{e^y} = \frac{(\cos x) \cancel{e^y} e^{\sin x}}{e^y} dx$

$\int e^{-y} dy = \int (\cos x) e^{\sin x} dx$ du

$u = \sin x$

$du = \cos x \, dx$

$-e^{-y} = \int e^u du$

$-e^{-y} = e^u + C$

$-e^{-y} = e^{\sin x} + C$

$-e^{-0} = e^{\sin 0} + C$

$-1 = 1 + C$

$-2 = C$

$y = 0, \ x = 0$

$-e^{-y} = e^{\sin x} - 2$

$e^{-y} = -e^{\sin x} + 2$

$+y = -\ln|-e^{\sin x} + 2|$

5) $dx \frac{\frac{dy}{dx}}{y+5} = \frac{(y+5)(x+2)}{y+5} dx$

$\int \frac{1}{y+5} dy = \int (x+2) dx$

$\ln|y+5| = \frac{1}{2}x^2 + 2x + C$

$\ln|1+5| = \frac{1}{2}(0)^2 + 2(0) + C$

$\ln 6 = C$

$y = 1, \ x = 0$

$\ln|y+5| = \frac{1}{2}x^2 + 2x + \ln 6$

$y + \cancel{5} = e^{\frac{1}{2}x^2 + 2x + \ln 6} - 5$

-5

$y = 6e^{\frac{1}{2}x^2 + 2x} - 5$

6.4

13) $y = Ce^{kt}$ $\quad$ $\begin{matrix} t & y \\ (0, & 50) \end{matrix}$ $\quad$ $\begin{matrix} t & y \\ (5, & 100) \end{matrix}$

$50 = Ce^{k(0)} \rightarrow C = 50$

$100 = Ce^{k(5)}$

$\frac{100}{50} = \frac{50e^{5k}}{50}$ $\qquad y = 50e^{\frac{\ln 2}{5}t}$

$\ln 2 = \ln e^{5k}$

$\frac{\ln 2}{5} = \frac{\cancel{5}k}{\cancel{5}}$

17) $A = Pe^{rt}$

$\frac{2898.44}{e^{.0525(30)}} = \frac{P\cancel{e^{.0525(30)}}}{\cancel{e^{.0525(30)}}}$

$600 = P$

$\frac{A}{P} = e^{rt} = 2$

$e^{.0525t} = 2$

$\ln e^{.0525t} = \ln 2$

$\frac{\cancel{.0525}t}{\cancel{.0525}} = \frac{\ln 2}{.0525}$

$t = 13.2$

23) $y = Ce^{kt}$

$y = 1e^{kt}$ $\longrightarrow$ $y = 1e^{\frac{\ln 2}{.5}t}$

$t =$ hour periods

$2 = 1e^{k(.5)}$ $\qquad y = e^{\frac{\ln 2}{.5}(24)}$

$\ln 2 = \ln e^{.5k}$ $\qquad y = e^{48 \ln 2}$

$\frac{\ln 2}{.5} = \frac{.5k}{.5}$

6.4

21 $y = Ce^{kt}$

$\frac{y}{C} = \frac{\cancel{C}e^{-0.0077t}}{\cancel{C}}$

$\frac{1}{2} = e^{-0.0077t}$

$\ln\frac{1}{2} = \ln e^{-.0077t}$

$\frac{\ln\frac{1}{2}}{-.0077} = \frac{-.0\cancel{077}t}{-.0\cancel{077}}$

35 $\frac{1}{2}$-half 5700 years

$y = Ce^{kt}$

$\frac{y}{c} = e^{kt} = \frac{1}{2}$

$e^{k(5700)} = \frac{1}{2}$

$\ln e^{5700k} = \ln\frac{1}{2}$

$\frac{\cancel{5700}k}{\cancel{5700}} = \frac{\ln\frac{1}{2}}{5700}$

$\frac{y}{c} = e^{kt}$

$\frac{y}{c} = e^{\frac{\ln\frac{1}{2}}{5700}t} = .445$

$\ln e^{\frac{\ln\frac{1}{2}}{5700}t} = \ln .445$

$\frac{\cancel{\frac{\ln\frac{1}{2}}{5700}}t}{\cancel{\frac{\ln\frac{1}{2}}{5700}}} = \frac{\ln .445}{\frac{\ln\frac{1}{2}}{5700}}$

$t = \underline{6658 \text{ years}}$

9 $\frac{dx\frac{dy}{\cancel{dx}}}{y^2} = \frac{-2x\cancel{y^2}}{\cancel{y^2}}dx$

$\int y^{-2}dy = \int -2x\,dx$

$-y^{-1} = -x^2 + C$

$y^{-1} = x^2 + C \longrightarrow \frac{1}{y} = x^2 + 3$

$(.25)^{-1} = 1^2 + C$

$4 = 1 + C$

$3 = C$

$\boxed{y = \frac{1}{x^2+3}}$

6.4

15 $\frac{A}{P} = \frac{Pe^{rt}}{P}$ $\rightarrow$ $\frac{2000}{1000} = \frac{1000e^{rt}}{1000}$

$2 = e^{.086t}$ $\quad 2 = e^{rt}$

$\ln 2 = \ln e^{.086t}$

$\frac{\ln 2}{.086} = \frac{\cancel{.086}t}{\cancel{.086}}$ $\quad A = 1000e^{.086(30)}$

$\frac{\ln 2}{.086} = t$

29 $\frac{y}{y_0} = \frac{\cancel{y_0}e^{-kt}}{\cancel{y_0}}$

$.05 = e^{-k(\frac{3}{k})}$

$.05 = e^{-3}$

36 $k = \frac{\ln\frac{1}{2}}{5700}$

$\frac{y}{C} = \frac{Ce^{\frac{\ln\frac{1}{2}}{5700}t}}{C}$

$\frac{.17}{e^{\frac{\ln\frac{1}{2}}{5700}}} = \frac{e^{\frac{\ln\frac{1}{2}}{5700}t}}{e^{\frac{\ln\frac{1}{2}}{5700}}}$ $\quad .18 = e^{\frac{\ln\frac{1}{2}}{5700}t}$ $\quad .16\, e^{\frac{\ln\frac{1}{2}}{5700}t}$

12,571 BC $\quad$ 12,101 BC $\quad$ 13,070 BC

6.4

30) $\ln|35-65| = k(10) + C$

$\ln 30 = \cancel{10k} + C$

$-10k \quad \cancel{-10k}$

$\ln 30 - 10k = C$

$\ln 30 - \cancel{10}\left(\frac{\ln\frac{1}{2}}{\cancel{10}}\right) = C$

$\ln \frac{30}{\frac{1}{2}} = C$

$\ln 60 = C$

$\ln|50-65| = k(20) + C$

$\ln 15 = 20k + C$

$\ln 15 = 20k + \ln 30 - 10k$

$-\ln 30 \qquad -\ln 30$

$\frac{\ln\frac{1}{2}}{10} = \frac{\cancel{10}k}{\cancel{10}}$

$\ln|y-65| = k(0) + \ln 60$

$\ln|y-65| = \ln 60$

$y - 65 = 60$
$+65 \quad +65$

$\cancel{y = 125}$

$y - \cancel{65} = -60$
$\cancel{+65} \quad +65$

$\boxed{y = 5°F}$

6.4

31 $\ln|90-20| = k(0) + C$

$\ln 70 = C$

$\ln|60-20| = k(10) + \ln 70$

$\ln 40 = 10k + \cancel{\ln 70}$

$-\ln 70 \qquad -\cancel{\ln 70}$

$\dfrac{\ln \frac{4}{7}}{10} = \dfrac{\cancel{10}k}{\cancel{10}}$

(a) $\ln|35-20| = \dfrac{\ln \frac{4}{7}}{10}t + \ln 70$

$\ln 15 = \dfrac{\ln \frac{4}{7}}{10}t + \cancel{\ln 70}$

$-\ln 70 \qquad -\cancel{\ln 70}$

$\ln \frac{3}{14} = \dfrac{\ln \frac{4}{7}}{10}t$

$\dfrac{10 \ln \frac{3}{14}}{\ln \frac{4}{7}} = t$

$27.527_{\text{min}} = t$

-10

17.527 minutes

(b) $\ln|90--15| = k(0) + C$

$\ln 105 = C$

$\ln|35--15| = \dfrac{\ln \frac{4}{7}}{10}t + \ln 105$

$\ln 50 = \dfrac{\ln \frac{4}{7}}{10}t + \cancel{\ln 105}$

$-\ln 105 \qquad -\cancel{\ln 105}$

$\ln \frac{10}{21} = \dfrac{\ln \frac{4}{7}}{10}t$

$\dfrac{10 \ln \frac{10}{21}}{\ln \frac{4}{7}} = \dfrac{\ln \frac{4}{7}\, t}{\ln \frac{4}{7}}$

$13.258_{\text{min}} = t$

6.4

32) $\ln|y-y_c| = kt + C$

$\ln|60| = k(0) + C$

$\ln 60 = C$

$\ln|70| = k(-20) + C$

$\ln 70 = -20k + \ln 60$

$-\ln 60 \qquad\qquad -\ln 60$

$\ln 70 - \ln 60 = -20k$

$\dfrac{\ln \frac{7}{6}}{-20} = \dfrac{\cancel{-20}k}{\cancel{-20}}$

(a) $\ln|y-y_c| = \underbrace{\dfrac{\ln\frac{7}{6}}{-20}(15) + \ln 60}_{\text{crap}}$

$y - y_c = e^{\text{crap}} \approx 53.449°$ ABOVE ROOM TEMPERATURE

(b) $\ln|y-y_c| = \dfrac{\ln\frac{7}{6}}{-20}(120) + \ln 60$

$y - y_c = e^{\text{crap2}} \approx 23.794°$ ABOVE

(c) $\ln|y-y_c|$

$\ln 10 = \dfrac{\ln\frac{7}{6}}{-20} t + \ln 60$

$-\ln 60 \qquad\qquad -\ln 60$

$\ln\frac{1}{6} = \dfrac{\ln\frac{7}{6}}{-20} t$

$\dfrac{-20\ln\frac{1}{6}}{\ln\frac{7}{6}} = \dfrac{\ln\frac{7}{6}\,t}{\ln\frac{7}{6}}$

$t = 232.469$ min from now