8.4

15) $\int_1^\infty \frac{5x+6}{x^2+2x}\,dx = \lim_{b\to\infty} \int_1^b \frac{5x+6}{x^2+2x}\,dx$

$$\lim_{b\to\infty} \int_1^b \frac{5x+5+1}{x^2+2x}\,dx$$

$$\lim_{b\to\infty} \int_1^b \frac{5(x+1)}{x^2+2x}\,dx + \lim_{b\to\infty} \int_1^b \frac{1}{x^2+2x}\,dx$$

$u = x^2+2x$

$du = 2x+2$

$= 2(x+1)$

$$\lim_{b\to\infty} \frac{5}{2}\int_{"1"}^b \frac{1}{u}\,du$$

$$\lim_{b\to\infty} \frac{5}{2}\left[\ln b - \ln(1^2+2(1))\right]$$

$x(x+2)$

$\frac{A}{x} + \frac{B}{x+2} = \frac{1}{x^2+2x}$

$A(x+2) + Bx = 1$

$x=-2 \quad -2B = 1$

$B = -\frac{1}{2}$

$x=0 \quad 2A = 1$

$A = \frac{1}{2}$

$$\lim_{b\to\infty} \int_1^b \left(\frac{1}{2x} + \frac{1}{2(x+2)}\right)dx$$

DIVERGE

$$\lim_{b\to\infty} \left[(2\ln b + 2\ln(b+2)) - (2\ln 1 + 2\ln(1+2))\right]$$

17) $\int_1^\infty xe^{-2x}\,dx$

$u = x \quad v = -\frac{1}{2}e^{-2x}$

$du = dx \quad dv = e^{-2x}\,dx$

$$\lim_{b\to\infty} \Big|_1^b -\frac{1}{2}xe^{-2x} - \int_1^b -\frac{1}{2}e^{-2x}\,dx$$

$$\lim_{b\to\infty} \Big|_1^b -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$$

$$\lim_{b\to\infty} \left[\left(-\frac{1}{2}be^{-2b} - \frac{1}{4}e^{-2b}\right) - \left(-\frac{1}{2}(1)e^{-2(1)} - \frac{1}{4}e^{-2(1)}\right)\right]$$

$$0 + \left(+\frac{1}{2e^2} + \frac{1}{4e^2}\right) \quad \frac{3}{4e^2}$$

8.4

15| PART DEMX

$$\int_1^\infty \frac{5x+6}{x^2+2x}\,dx$$

$x(x+2)$

$$\frac{A}{x}+\frac{B}{x+2}=\frac{5x+6}{x^2+2x}$$

$$A(x+2)+Bx=5x+6$$

$x=0 \quad 2A=6 \quad A=3$

$x=-2 \quad -2B=-4 \quad B=2$

$$\lim_{b\to\infty}\int_1^b\left(\frac{3}{x}+\frac{2}{x+2}\right)dx$$

$$\lim_{b\to\infty}\Big|_1^b\ 3\ln|x|+2\ln|x+2|$$

$$\lim_{b\to\infty}\left[(3\ln b+2\ln(b+2))-(3\ln 1+2\ln(1+2)\right]$$

$\boxed{\text{DIVERGES}}$

37| $$\int_0^\infty \frac{ds}{(1+s)\sqrt{s}}$$

$$\lim_{b\to\infty} 2\int_0^b \frac{1}{1+(\sqrt{s})^2}\cdot\frac{1}{\sqrt{s}}\,ds\ \frac{1}{2}$$

$u=s^{1/2}$

$du=\frac{1}{2}s^{-1/2}\,ds$

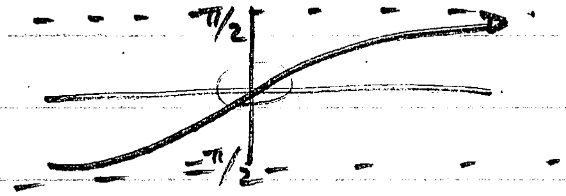
$$\lim_{b\to\infty} 2\Big|_0^b \tan^{-1}\sqrt{s}$$

$$\lim_{b\to\infty} 2\left[\tan^{-1}\sqrt{b}-\tan^{-1}\sqrt{0}\right]$$

$$2\left[\frac{\pi}{2}-0\right]=\pi$$

$$\int\frac{1}{1+u^2}\,du=\tan^{-1}u$$

FORMULA



19| $$\int_1^\infty x\ln(x)\,dx$$

$u=\ln x \quad v=\frac{1}{2}x^2$

$du=\frac{1}{x}dx \quad dv=x\,dx$

$$\lim_{b\to\infty}\Big|_1^b \frac{1}{2}x^2\ln(x)-\int\frac{1}{2}x^2\cdot\frac{1}{x}\,dx$$

$$\lim_{b\to\infty}\Big|_1^b \frac{1}{2}x^2\ln(x)-\int\frac{1}{2}x\,dx$$

$$\lim_{b\to\infty}\Big|_1^b \frac{1}{2}x^2\ln(x)-\frac{1}{4}x^2=\lim_{b\to\infty}\left[\frac{1}{2}b^2\ln(b)-\frac{1}{4}b^2\right]-$$

$\boxed{\text{DIVERGES}}$

8.4

25 $\int_0^2 \frac{dx}{1-x^2} = \int_0^1 \frac{dx}{1-x^2} + \int_1^2 \frac{dx}{1-x^2}$

$$\lim_{b\to 1^-} \int_0^b \frac{dx}{(1-x)(1+x)} + \lim_{a\to 1^+} \int_a^2 \frac{dx}{(1-x)(1+x)}$$

$$\frac{A}{1-x} + \frac{B}{1+x} = \frac{1}{1-x^2}$$

$$A(1+x) + B(1-x) = 1$$

$x=1 \quad 2A = 1 \to A = 1/2$

$x=-1 \quad 2B = 1 \to B = 1/2$

$$\lim_{b\to 1^-} \int_0^b \left(\frac{1}{2}\cdot\frac{1}{1-x} + \frac{1}{2}\cdot\frac{1}{1+x}\right)dx + \lim_{a\to 1^+} \int_a^2 \left(\frac{1}{2}\cdot\frac{1}{1-x} + \frac{1}{2}\cdot\frac{1}{1+x}\right)dx$$

$$\lim_{b\to 1^-} \Big|_0^b -\frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x| + \lim_{a\to 1^+} \Big|_a^2 -\frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x|$$

$u = 1-x$

$du = -1$

$$\lim_{b\to 1^-} \Big|_0^b \ln(1-x)^{-1/2} + \ln(1+x)^{1/2} + \lim_{a\to 1^+} \Big|_a^2 \ln(1-x)^{-1/2} + \ln(1+x)^{1/2}$$

$$\lim_{b\to 1^-} \Big|_0^b \ln(1-x)^{-1/2}(1+x)^{1/2} + \lim_{a\to 1^+} \Big|_a^2 \ln(1-x)^{-1/2}(1+x)^{1/2}$$

$$\lim_{b\to 1^-} \Big|_0^b \ln\frac{(1+x)^{1/2}}{(1-x)^{1/2}} + \lim_{a\to 1^+} \Big|_a^2 \frac{(1+x)^{1/2}}{(1-x)^{1/2}}$$

$$\lim_{b\to 1^-} \left[\ln\frac{(1+b)^{1/2}}{(1-b)^{1/2}} - \ln\frac{(1+0)^{1/2}}{(1-0)^{1/2}}\right] + \lim_{a\to 1^+} \left[\ln\frac{(1+2)^{1/2}}{(1-2)^{1/2}} - \ln\frac{(1+a)^{1/2}}{(1-a)^{1/2}}\right]$$

$\boxed{\text{DIVERGES}}$

8.4

27 $\frac{1}{2}\int_0^1 \frac{(x+1)^2}{\sqrt{x^2+2x}}\,dx$ du

$u = x^2 + 2x$

$du = (2x+2)\,dx$

$\frac{1}{2}\int_0^3 (u)^{-1/2}\,du$

$\frac{1}{2}\lim_{a \to 0^+} \int_a^3 u^{-1/2}\,du$

$\frac{1}{2}\lim_{a \to 0^+} \Big|_a^3 2u^{1/2}$

$\lim_{a \to 0^+} \left[2(3)^{1/2} - 2a^{1/2} \right] = \boxed{\sqrt{3}}$

20 $\int_0^\infty (x+1)e^{-x}\,dx$

$\lim_{b \to \infty} \int_0^b (x+1)e^{-x}\,dx$

$u = x+1 \qquad v = -e^{-x}$

$du = dx \qquad dv = e^{-x}\,dx$

$\lim_{b \to \infty} \Big|_0^b -e^{-x}(x+1) - \int_0^b -e^{-x}\,dx$

$\lim_{b \to \infty} \Big|_0^b -e^{-x}(x+1) - e^{-x}$

$\lim_{b \to \infty} \left[\underbrace{-e^{-b}(b+1)}_{0} - \underbrace{e^{-b}}_{0} \right] - \left[-e^{-0}(0+1) - e^{-0} \right]$

$1 + 1 = \boxed{2}$