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#17 $\sum_{n=0}^{\infty} \left(\sin\left(\frac{\pi}{4} + n\pi\right)\right)^n$

$\left(\sin\frac{\pi}{4}\right)^0$ $\left(\sin\left(\frac{\pi}{4}+\pi\right)\right)^1$ $\left(\sin\left(\frac{\pi}{4}+2\pi\right)\right)^2$

1 $-1/\sqrt{2}$ $\left(1/\sqrt{2}\right)^2$

$\left(\sin\left(\frac{\pi}{4}+3\pi\right)\right)^3$

$\left(-1/\sqrt{2}\right)^3$

$\sum_{n=0}^{\infty} \left(-\frac{1}{\sqrt{2}}\right)^n$

$\left|-\frac{1}{\sqrt{2}}\right| < 1$?

CONVERGES

$$\frac{1}{1-\left(-\frac{1}{\sqrt{2}}\right)}$$

$$\frac{1}{1+\frac{1}{\sqrt{2}}}$$

$$\frac{1}{\frac{\sqrt{2}+1}{\sqrt{2}}} = \boxed{\frac{\sqrt{2}}{\sqrt{2}+1}} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= \frac{2-\sqrt{2}}{2-1} = \boxed{2-\sqrt{2}}$$

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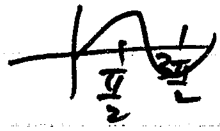
25] $$\sum_{n=0}^{\infty} \sin^n x = \sum_{n=0}^{\infty} (\sin x)^n$$



$$|\sin x| < 1 \qquad x \neq \frac{\pi}{2} + \pi k$$

$$\boxed{f(x) = \frac{1}{1-\sin x}} \cdot \frac{1+\sin x}{1+\sin x} = \frac{1+\sin x}{1-\sin^2 x}$$

$$= \frac{1+\sin x}{\cos^2 x}$$

21] $$\sum_{n=0}^{\infty} 2^n x^n = \boxed{\sum_{n=0}^{\infty} (2x)^n}$$

$$|2x| < 1$$

$$2x < 1 \qquad 2x > -1$$

$$x < \frac{1}{2} \qquad x > \frac{-1}{2}$$

$$\boxed{\left(-\frac{1}{2}, \frac{1}{2}\right)}$$

$$\boxed{f(x) = \frac{1}{1-2x}} \rightarrow (1-2x)^{-1}$$

$2^n n x^{n-1}$

$2 \cdot 2^{n-1} n x^{n-1}$

$\underline{2n(2x)^{n-1}}$

27] $$\sum_{n=0}^{\infty} n(2x)^{n-1}[2] = \boxed{\sum_{n=0}^{\infty} 2n(2x)^{n-1}}$$

$$\boxed{\left(-\frac{1}{2}, \frac{1}{2}\right)}$$

$$f'(x) = -(1-2x)^{-2}[-2]$$

$$\boxed{f'(x) = \frac{2}{(1-2x)^2}}$$

$$\sum_{n=0}^{\infty} (2x)^n = 1 + 2x + (2x)^2 + (2x)^3 + \ldots$$

$$= 1 + 2x + 4x^2 + 8x^3$$

$$= 0 + 2 + 8x + 24x^2$$

0 1 2 3

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27| $\sum_{n=0}^{\infty} (2x)^n$ $(-\frac{1}{2}, \frac{1}{2})$ $f(x) = -\frac{1}{2} \frac{1}{1-2x}$ -2

$u = 1-2x$

$du = -2dx$

31| $\sum_{n=0}^{\infty} \frac{1}{2(n+1)} (2x)^{n+1} = \boxed{\sum_{n=0}^{\infty} \frac{1}{2(n+1)} (2x)^{n+1}}$

$\boxed{(-\frac{1}{2}, \frac{1}{2})}$ $\boxed{\int f(x) = -\frac{1}{2} \ln |1-2x|}$

51| $\sum_{n=1}^{\infty} \frac{\pi}{2} \left(\frac{1}{2}\right)^n$

$\frac{1}{2} \pi r^2$ ← AREA OF EACH

$\sum_{n=1}^{\infty} \frac{\pi}{2} \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-1}$

$\underbrace{2^n}_{\text{HOW MANY}} \cdot \frac{1}{2}\pi \left(\frac{1}{2^n}\right)^2$

$\frac{\pi}{2} \left(\frac{1}{2^{2n}}\right) \cdot 2^n$

$\sum_{n=1}^{\infty} \frac{\pi}{4} \left(\frac{1}{2}\right)^{n-1}$

$\frac{2^n}{2^{2n}} = 2^{n-2n} = \frac{1}{2^n}$

$\frac{a}{1-r}$ $\frac{\frac{\pi}{4}}{1 - \frac{1}{2}} = \boxed{\frac{\pi}{2}}$

19| $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

$\frac{\frac{e}{\pi}}{1 - \frac{e}{\pi}} = \frac{\frac{e}{\pi}}{\frac{\pi - e}{\pi}}$

$\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)\left(\frac{e}{\pi}\right)^{n-1}$

$\frac{e}{\cancel{\pi}} \cdot \frac{\cancel{\pi}}{\pi - e} = \frac{e}{\pi - e}$

54| $f(x) = \frac{1}{1+3x} = \frac{1}{1-(-3x)}$ $1 \to a$ $r = -3x$

$\sum_{n=0}^{\infty} (-3x)^n$ $|-3x| < 1$

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45) $1.\overline{414}$

$1 + .\overline{414}$

$\frac{999}{999} + \frac{414}{999}$

$\boxed{\frac{1413}{999}} = \boxed{\frac{157}{111}}$

$$\sum_{n=0}^{\infty} .414\,(.001)^n$$

$$\frac{.414}{1-.001}$$

$$\frac{\frac{414}{1000}}{\frac{999}{1000}} = \frac{414}{999}$$