9.2

11 $f(x) = (1+x^2)^{1/2}$

$f'(x) = \frac{1}{2}(1+x^2)^{-1/2}[2x] = x(1+x^2)^{-1/2}$ (f, g)

$f''(x) = (1+x^2)^{-1/2} + -\frac{1}{2}(1+x^2)^{-3/2}(2x)\cdot x$

$f''(x) = (1+x^2)^{-1/2} - x^2(1+x^2)^{-3/2}$ (f, g)

$f'''(x) = -\frac{1}{2}(1+x^2)^{-3/2}[2x] - 2x(1+x^2)^{-3/2} + \frac{3}{2}(1+x^2)^{-5/2}[2x]x^2$

$= -x(1+x^2)^{-3/2} - 2x(1+x^2)^{-3/2} + 3(1+x^2)^{-5/2}x^3$

$= -3x(1+x^2)^{-3/2} + 3x^3(1+x^2)^{-5/2}$ (f, g, f, g)

$f^4(x) = -3(1+x^2)^{-3/2} + \frac{+3}{2}(1+x^2)^{-5/2}(2x)[2x]$

$+ 9x^2(1+x^2)^{-5/2} + \frac{-5}{2}(1+x^2)^{-7/2}[2x](3x^3)$

$f(0) = 1$

$f'(0) = 0$

$f''(0) = 1$

$f'''(0) = 0$

$f^4(0) = -3$

$P_4(x) = 1 + \frac{1}{2}x^2 - \frac{3}{24}x^4$

$\boxed{P_4(x) = 1 + \frac{1}{2}x^2 - \frac{1}{8}x^4}$

11 $\frac{x}{1-x^3}$

use $\frac{1}{1-x} = 1 + x + x^2 + \ldots + x^n + \ldots$

$\frac{1}{1-x^3} = 1 + x^3 + x^6 + \ldots + x^{3n} + \ldots$

$\frac{x}{1}\cdot\frac{1}{1-x^3} = x + x^4 + x^7 + \ldots + x^{3n+1} + \ldots$

9.2

13) $\frac{1}{x+1}$, $x=2$

$f(x) = (x+1)^{-1}$ $\quad f(2) = \frac{1}{3}$

$f'(x) = -(x+1)^{-2}$ $\quad f'(2) = -\frac{1}{9}$

$f''(x) = 2(x+1)^{-3}$ $\quad f''(2) = \frac{2}{27}$

$f'''(x) = -6(x+1)^{-4}$ $\quad f'''(2) = \frac{-2}{27}$

$f^4(x) = 24(x+1)^{-5}$ $\quad f^4(2) =$

$$\frac{1}{3} - \frac{1}{9}(x-2) + \frac{2}{27}\cdot\frac{1}{2}(x-2)^2 - \frac{2}{27}\cdot\frac{1}{6_3}(x-2)^3$$

$$+ \dots + (-1)^n \frac{1}{3^{n+1}}(x-2)^n$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}}(x-2)^n$$

$\frac{1}{3}$ $\quad \frac{1}{9}$ $\quad \frac{1}{27}$ $\quad \frac{1}{81}$

0 $\quad$ 1 $\quad$ 2 $\quad$ 3

19) $f(x) = \sin x$ $\quad f(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ $\quad P_1(x) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}(x-\frac{\pi}{4})$

$f'(x) = \cos x$ $\quad f'(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

$f''(x) = -\sin x$ $\quad f''(\frac{\pi}{4}) = -\frac{1}{\sqrt{2}}$ $\quad P_2(x) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}(x-\frac{\pi}{4}) - \frac{1}{\sqrt{2}}\frac{1}{2}(x-\frac{\pi}{4})^2$

$f'''(x) = -\cos x$ $\quad f'''(\frac{\pi}{4}) = -\frac{1}{\sqrt{2}}$

~~$f^4(x) = \sin x$~~ $\quad$ ~~$f^4(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$~~

$$P_2(x) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}(x-\frac{\pi}{4}) - \frac{1}{2\sqrt{2}}(x-\frac{\pi}{4})^2$$

$$P_3(x) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}(x-\frac{\pi}{4}) - \frac{1}{2\sqrt{2}}(x-\frac{\pi}{4})^2 - \frac{1}{6\sqrt{2}}(x-\frac{\pi}{4})^3$$