9.4

51| $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} = \frac{A}{n^2} + \frac{B}{(n+1)^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$An^2 + B(n+1)^2 = 2n+1$ $\quad A(n+1)^2 + Bn^2 = 2n+1$

$n=0 \quad A=1$

$n=-1 \quad B=-1$

$\left(1 - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{9}\right) + \ldots \quad - \frac{1}{(n+1)^2}$

53| $\sum_{n=1}^{\infty} \left(\frac{1}{\ln(n+2)} - \frac{1}{\ln(n+1)}\right)$

$\left(\frac{1}{\ln 3} - \frac{1}{\ln 2}\right) + \left(\frac{1}{\ln 4} - \frac{1}{\ln 3}\right) + \left(\frac{1}{\ln 5} - \frac{1}{\ln 4}\right)$

$-\frac{1}{\ln 2} + \frac{1}{\ln(n+2)}$

27| $\left|\frac{x^2-1}{3}\right| < 1$

$|x^2-1| < 3$

$|x^2| < 4$

$|x| < 2$

$(-2, 2)$

$\frac{1}{1 - \frac{x^2-1}{3}}$

$\frac{1}{\frac{3}{3} - \frac{x^2-1}{3}}$

$\frac{3}{3-(x^2-1)}$

$\frac{3}{4-x^2}$

9.4

25 $\sum_{n=0}^{\infty} \left(\frac{\sqrt{x}}{2} - 1\right)^n$

$\frac{1}{1 - \left(\frac{\sqrt{x}}{2} - 1\right)}$

$\left|\frac{\sqrt{x}}{2} - 1\right| < 1$

$\frac{1}{2 - \frac{\sqrt{x}}{2}}$

$\left|\frac{\sqrt{x}}{2}\right| < 2$

$\frac{2}{4 - \sqrt{x}}$

$|\sqrt{x}| < 4$

$|x| < 16$

$(-16, 16)$ $\boxed{(0, 16)}$

23 $\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{4^n} = \sum_{n=0}^{\infty} \left(\frac{(x-1)^2}{4}\right)^n$

$\left|\frac{(x-1)^2}{4}\right| < 1$