9.5

55) $\sum_{n=1}^{\infty} \frac{n^n (x+2)^n}{3^n n!}$

$\lim_{n\to\infty} \left| \frac{(n+1)^{n+1} \cancel{(x+2)^{n+1}}}{\cancel{3^{n+1}} \cancel{(n+1)!}} \cdot \frac{\cancel{3^n} \cancel{n!}}{n^n \cancel{(x+2)^n}} \right| < 1$

$\lim_{n\to\infty} \left| \frac{(n+1)^{\cancel{n+1}}}{\cancel{n+1} \cdot 3 \ n^n} \ x+2 \right| < 1$

$\frac{1}{3} \lim_{n\to\infty} \left| \frac{(n+1)^n}{n^n} \ x+2 \right| < 1$

$\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n$

$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = L$

$\ln \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = \ln L$

$\lim_{n\to\infty} n \ln\left(1+\frac{1}{n}\right) = \ln L$

$\lim_{n\to\infty} \frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}} = \ln L$

$\lim_{n\to\infty} \frac{\frac{1}{1+\frac{1}{n}} \cancel{\left[-\frac{1}{n^2}\right]}}{\cancel{\left[-\frac{1}{n^2}\right]}} = \ln L$

$1 = \ln L$

$e = L$

$\cancel{\lim_{n\to\infty} \left(\frac{n}{e}\right)^n \frac{1}{n!}}$

$\lim_{n\to\infty} \left| \frac{(n+1)^{n+1}}{\cancel{(n+1)!}} \cdot \frac{n!}{n^n} \right| < 1$

$e < 1$ D1

$\frac{1}{3} \cancel{\lim_{n\to\infty}} \ e \ |x+2| < 1$

$|x+2| < \frac{3}{e}$

$-2 + \frac{-3}{e}, \quad -2 + \frac{3}{e}$

$\sum_{n=1}^{\infty} \frac{n^n (\cancel{-2} - \frac{3}{e} + \cancel{2})^n}{3^n n!}$

$\sum_{n=1}^{\infty} \frac{n^n \left(-\frac{3}{e}\right)^n}{3^n n!}$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \left(\frac{n \frac{3}{e}}{3}\right)^n, , ,$

$\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n! \ e^n} < \frac{n^n}{n!}$ D1

$\lim_{n\to\infty} \frac{n^n}{n! \ e^n}$ $\quad \lim_{n\to\infty} \left| \frac{(n+1)^{n+1}}{\cancel{(n+1)!} \ e^{n+1}} \cdot \frac{n! \ e^n}{n^n} \right| < 1$

$\frac{1}{e} \lim_{n\to\infty} \left| \left(\frac{n+1}{n}\right)^n \right| < 1$

$\frac{1}{e} \cdot e = 1$

$$\lim_{n\to\infty} \frac{n^n}{n!\, e^n} \qquad \frac{n \cdot n \cdot n \cdot n \cdot n \cdot n}{(n)(n-1)(n-2)(n-3)(n-4)}$$

$$\lim_{n\to\infty} \frac{n^n}{n!} \cdot \lim_{n\to\infty} \frac{1}{e^n} \qquad \frac{(n+1)^{n+1}}{(n+1)!\, e^{n+1}} < \frac{n^n}{n!\, e^n}$$

$$e \cdot 0 = 0 \qquad \frac{\cancel{n!}\, \cancel{e^n}\, (n+1)^{\cancel{n+1}\, n}}{\cancel{(n+1)}!\, \cancel{e^{n+1}}\, e\, n^n} < 1$$

$$\left(\frac{n+1}{n}\right)^n < e$$

FOR ALL FINITE n

9.5

37 $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$ $\lim_{n\to\infty} \left| \frac{(4x+1)^{n+1}}{(4x+1)^n} \right| < 1$

$x = -\frac{1}{2}$ $\sum_{n=0}^{\infty} (-1)^n (4(-\frac{1}{2})+1)^n$ $\lim_{n\to\infty} |4x+1| < 1$

$\sum_{n=0}^{\infty} (-1)^n (-1)^n$ $4x+1<1$ $4x+1>-1$

$4x<0$ $4x>-2$

$\sum_{n=0}^{\infty} 1$ DIVERGE $x<0$ $x>-\frac{1}{2}$

$(-\frac{1}{2}, 0)$

$x=0$ $\sum_{n=0}^{\infty} (-1)^n (4(0)+1)^n$

$\sum_{n=0}^{\infty} (-1)^n (1)^n$

$\sum_{n=0}^{\infty} (-1)^n$ DIVERGE

9.5

48 $\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$

$$\lim_{n\to\infty} \left| \frac{(4x-5)^{2n+3}}{(n+1)^{3/2}} \cdot \frac{n^{3/2}}{(4x-5)^{2n+1}} \right| < 1$$

$(4x-5)^2$

$$\lim_{n\to\infty} \left| \frac{n^{3/2}}{(n+1)^{3/2}} (4x-5)^2 \right| < 1$$

$$|(4x-5)^2| < 1$$

$$|4x-5| < 1$$

$4x-5<1$ $\qquad$ $4x-5>-1$

$4x<6$ $\qquad$ $4x>4$

$x<\frac{3}{2}$ $\qquad$ $x>1$

$\left[1, \frac{3}{2}\right]$

$x=1$ $\quad \sum_{n=1}^{\infty} \frac{(4(1)-5)^{2n+1}}{n^{3/2}}$

$\sum_{n=1}^{\infty} \frac{(-1)^{2n+1}}{n^{3/2}}$

$\sum_{n=1}^{\infty} \frac{-1}{n^{3/2}}$ CONVERGE

$\sum_{n=1}^{\infty} \frac{(4(\frac{3}{2})-5)^{2n+1}}{n^{3/2}}$

$\sum_{n=1}^{\infty} \frac{(1)^{2n+1}}{n^{3/2}}$

$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

RATIO TEST
LIMIT COMPARISON
DIRECT COMPARISON
p-SERIES
GEOMETRIC
TELESCOPING SERIES
INTEGRAL TEST
ALTERNATING SERIES
nth-Term Test

9.5

23 $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$ $\qquad$ (DI) $\sum_{n=1}^{\infty} \frac{1+n}{n^2}$ $\qquad$ $\sum_{n=1}^{\infty} \frac{1}{n}$ DI

$\lim_{n\to\infty} \frac{1+n}{n^2} = 0$

$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1+n}{n^2}} = \lim_{n\to\infty} \frac{1}{n} \cdot \frac{n^2}{1+n} = 1$

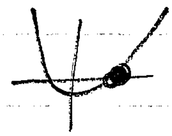
$n^2(\cancel{n+1})^2 \quad \frac{1+(n+1)}{(\cancel{n+1})^2} < \frac{1+n}{\cancel{n^2}} \cancel{n^2}(n+1)^2$

$n^2(n+2) < (1+n)(n+1)^2$

$\cancel{n^3}+2n^2 < \cancel{n^3}+3n^2+3n+1$

$0 < n^2+3n+1$

FOR $n > 1$



25 $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \ln n}$ $\qquad$ DIVERGES $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ $\quad \lim_{b\to\infty} \int_2^b \frac{1}{x \ln x}\, dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$\lim_{n\to\infty} \frac{1}{n \ln n} = 0$ $\qquad$ $\lim_{b\to\infty} \Big]_2^b \ln(\ln x)$

$\frac{1}{(n+1)\ln(n+1)} < \frac{1}{n \ln n}$ $\qquad$ $\lim_{b\to\infty} [\ln(\ln b) - \ln(\ln 2)] = \infty$

FOR ALL n $\qquad$ DIVERGES

$\sum_{n=1}^{\infty} \frac{1}{n-1}$ $\qquad$ $\frac{1}{n} < \frac{1}{n-1}$ $\qquad$ $\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n-1}}$ $\quad \lim_{n\to\infty} \frac{n-1}{n} = 1$

DI

LIMIT

COMPARISON

$\sum \frac{1}{n^2+1}$ $\qquad$ $\frac{1}{n^2+1} < \frac{1}{n^2}$

CON

DIRECT

COMPARISON