3.7

53) $v = 8\sqrt{s-t} + 1$

$\frac{dv}{dt} = 8(s-t)^{1/2} + 1$

$= 4(s-t)^{-1/2}\left[\frac{ds}{dt} - 1\right]$

$= 4(s-t)^{-1/2}\left[8\sqrt{s-t} + 1 - 1\right]$

$= 32(s-t)^{-1/2}\sqrt{s-t}$

$= 32$

37) $y = x\sqrt{x^2+1}$

$y = x(x^2+1)^{1/2}$

$\frac{dy}{dx} = (x^2+1)^{1/2} + \frac{1}{2}(x^2+1)^{-1/2}[2x]\cdot x$

$\frac{dy}{dx} = (x^2+1)^{1/2} + x^2(x^2+1)^{-1/2}$

$\frac{d^2y}{dx^2} =$

45) $y^4 = y^2 - x^2$

$4y^3\frac{dy}{dx} = 2y\frac{dy}{dx} - 2x$

$-2y\frac{dy}{dx} \quad -2y\frac{dy}{dx}$

$\frac{\frac{dy}{dx}(4y^3-2y)}{(4y^3-2y)} = \frac{-2x}{4y^3-2y}$

$\left(\overset{x}{\frac{\sqrt{3}}{4}}, \overset{y}{\frac{\sqrt{3}}{2}}\right)$

55) (a) $x^3 + y^3 - 9xy = 0$

$3x^2 + 3y^2\frac{dy}{dx} - 9y - 9x\frac{dy}{dx} = 0$

$-3x \qquad +9y \qquad -3x^2 + 9y$

$\frac{\frac{dy}{dx}(3y^2-9x)}{(3y^2-9x)} = \frac{-3x^2+9y}{3y^2-9x}$

3.7

55) $-3x^2 + 9y = 0$

$9y = 3x^2$

$y = \frac{x^2}{3}$

$y = \frac{(\sqrt[3]{54})^2}{3}$

$y = \frac{(\sqrt[3]{27 \cdot 2})^2}{3}$

$y = \frac{(3\sqrt[3]{2})^2}{3}$

$y = \frac{9\sqrt[3]{4}}{3}$

$y = 3\sqrt[3]{4}$

$\frac{dy}{dx} = \frac{-3x^2 + 9y}{3y^2 - 9x} = 0$

$x^3 + y^3 - 9xy = 0$

$x^3 + (\frac{x^2}{3})^3 - 9x(\frac{x^2}{3}) = 0$

$x^3 + \frac{1}{27}x^6 - 3x^3 = 0$

$\frac{1}{27}x^6 - 2x^3 = 0$

$x^3(\frac{1}{27}x^3 - 2) = 0$

$x^3 = 0 \qquad \frac{1}{27}x^3 - 2 = 0$

$x = 0 \qquad +2 \quad +2$

$\frac{1}{27}x^3 = 2$

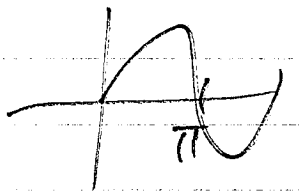
$x^3 = 54$

$x = \sqrt[3]{54}$

33) $y = \sqrt[3]{x}$

$y = x^{1/3}$

$y' = \frac{1}{3}x^{-2/3}$



47) $x^3y^2 = \cos(\pi y)$

$(-1)^3(1)^2 = \cos(\pi \cdot 1)$

$-1 = -1$ ✓

$x^3y^2 = \cos(\pi y)$

$3x^2y^2 + 2x^3y\frac{dy}{dx} = -\sin(\pi y)[\pi \frac{dy}{dx}]$

$3(-1)^2(1)^2 + 2(-1)^3(1)\frac{dy}{dx} = -\sin(\pi \cdot 1)[\pi \frac{dy}{dx}]$

$3 - 2\frac{dy}{dx} = 0$

$\frac{3}{2} = \frac{2\frac{dy}{dx}}{2}$

$\frac{3}{2} = \frac{dy}{dx}$

3.7 #7, 8

7) $x + \tan(xy) = 0$

$1 + \sec^2(xy)\left[y + x\frac{dy}{dx}\right] = 0$

$1 + y\sec^2(xy) + x\sec^2(xy)\frac{dy}{dx} = 0$

$-1 \quad -y\sec^2(xy) \qquad\qquad -1 - y\sec^2(xy)$

$$\frac{x\sec^2(xy)\frac{dy}{dx}}{x\sec^2(xy)} = \boxed{\frac{-1 - y\sec^2(xy)}{x\sec^2(xy)} = \frac{dy}{dx}}$$

$$\frac{-1}{x\sec^2(xy)} - \frac{y\sec^2(xy)}{x\sec^2(xy)}$$

$$\boxed{-\frac{1}{x}\cos^2(xy) - \frac{y}{x} = \frac{dy}{dx}}$$

11) $(x-1)^2 + (y-1)^2 = 13$

$2(x-1)[1] + 2(y-1)\left[\frac{dy}{dx}\right] = 0$

$-2x+2 \qquad\qquad -2x+2$

$$\frac{(2y-2)\frac{dy}{dx}}{2y-2} = \boxed{\frac{-2x+2}{2y-2} = \frac{dy}{dx}}$$

$$-\frac{x-1}{y-1} \qquad \frac{2(-x+1)}{2(y-1)} = \frac{-x+1}{y-1}$$

$$\frac{x-1}{-y+1} \qquad \frac{3-1}{-4+1} = -\frac{2}{3}$$

3.7 f & g

13) $x^2y - xy^2 = 4$

$2xy + x^2\frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} = 0$

$-2xy \qquad +y^2 \qquad (2y\frac{dy}{dx})(-x)$

$-2xy + y^2$

$$\frac{\frac{dy}{dx}(x^2-2xy)}{x^2-2xy} = \boxed{\frac{-2xy+y^2}{x^2-2xy} = \frac{dy}{dx}}$$

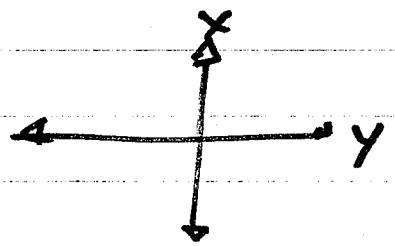
$x^2 - 2xy = 0$

$x(x-2y) = 0$

$x = 0 \qquad x - 2y = 0$

$x = 2y$

EVERYWHERE EXCEPT $x=0,\ x=2y$



f & g

55) $x^3 + y^3 - 9xy = 0$

$3x^2 + 3y^2\frac{dy}{dx} - 9y - 9x\frac{dy}{dx} = 0$

$-3x^2 \qquad -9y \qquad -3x^2 + 9y$

$$\frac{\frac{dy}{dx}(3y^2-9x)}{3y^2-9x} = \frac{-3x^2+9y}{3y^2-9x}$$

$$= \frac{3(-x^2+3y)}{3(y^2-3x)}$$

$$= \frac{-x^2+3y}{y^2-3x}$$

(a)

@ (4,2)

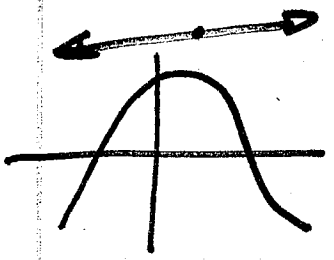
$\frac{-(4)^2+3(2)}{(2)^2-3(4)} = \frac{-10}{-8} = \frac{5}{4}$

@ (2,4)

$\frac{-(2)^2+3(4)}{(4)^2-3(2)} = \frac{8}{10} = \frac{4}{5}$

3.7

55) (b) $\frac{-x^2+3y}{y^2-3x}$



$-x^2+3y=0$

$3y=x^2$ ✓

$y=\frac{x^2}{3}$

$x^3+y^3-9xy=0$ ✓

$x^3+\left(\frac{x^2}{3}\right)^3-9x\left(\frac{x^2}{3}\right)=0$

$x^3+\frac{x^4}{27}-3x^3=0$

$\frac{1}{27}x^4-2x^3=0$

$x^3\left(\frac{1}{27}x^3-2\right)=0$

$x^3=0$ $\qquad$ $\frac{1}{27}x^3-2=0$

$x\neq 0$ $\qquad$ $\frac{1}{27}x^3=2$

ORIGIN $\qquad$ $x^3=54$

$x=\sqrt[3]{54}$

$\sqrt[3]{54}+y^3-9\sqrt[3]{54}\,y=0$

3.7

25) $y = 2\sin(\pi x - y)$

x y
(1,0) =

$\frac{dy}{dx} = 2\cos(\pi x - y)\left[\pi - \frac{dy}{dx}\right]$

$\frac{dy}{dx} = 2\pi\cos(\pi x - y) - 2\cos(\pi x - y)\frac{dy}{dx}$

$+ 2\cos(\pi x - y)\frac{dy}{dx}$

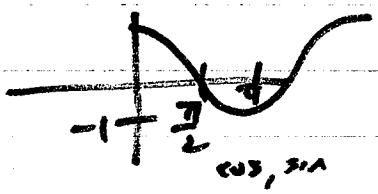
$+ 2\cos(\pi x - y)\frac{dy}{dx}$

$\frac{dy}{dx}(1 + 2\cos(\pi x - y)) = 2\pi\cos(\pi x - y)$

$\frac{dy}{dx} = \frac{2\pi\cos(\pi x - y)}{1 + 2\cos(\pi x - y)}$

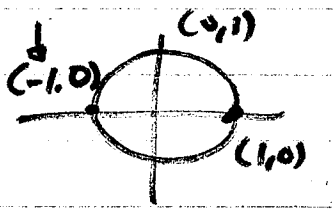
$\cos(\pi \cdot 1 - 0) = -1$

$\cos(\pi \cdot 1 - 0) = -1$



TANGENT $\frac{dy}{dx} = \frac{2\pi(-1)}{1 + 2(-1)}$

$= \frac{-2\pi}{-1} = 2\pi = m$



$y = 2\pi(x - 1)$

NORMAL $m = -\frac{1}{2\pi}$ $\quad y = -\frac{1}{2\pi}(x - 1)$

3.7

21) $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$ $\quad (-1, 0)$

$12x + 3y + 3x\frac{dy}{dx} + 4y\frac{dy}{dx} + 17\frac{dy}{dx} = 0$

$-12x \quad -3y$

$-12x-3y$

$\frac{dy}{dx}(3x + 4y + 17) = -12x - 3y$

$\frac{dy}{dx} = \frac{-12x-3y}{3x+4y+17}$

$3x+4y+17$

TANGENT

$\frac{dy}{dx} = \frac{-12(-1) - 3(0)}{3(-1) + 4(0) + 17} = \frac{12}{14} = \frac{6}{7}$ $\quad y = \frac{6}{7}(x+1)$

NORMAL $\quad m = -\frac{7}{6}$ $\quad y = -\frac{7}{6}(x+1)$

3.7

38) $x^2 + y^2 = 25$ $(3,-4)$

$2x + 2y\frac{dy}{dx} = 0$

$-2x \qquad -2x$

$\frac{2y\frac{dy}{dx}}{2y} = \frac{-2x}{2y}$

$\frac{dy}{dx} = -\frac{x}{y}$

TANGENT: $m = \frac{-3}{-4} = \frac{3}{4}$

$y + 4 = \frac{3}{4}(x-3)$

NORMAL: $m = -\frac{4}{3}$

$y + 4 = -\frac{4}{3}(x-3)$