3.8

29) $f(x) = \cos x + 3x$

(a) $f'(x) = -\sin x + 3$

(b) $f(0) = 1$

$f'(0) = 3$

$$\begin{array}{ccc} f & & f^{-1} \\ (0,1) & \rightarrow & (1,0) \end{array}$$

(c) $f^{-1}(1) = 0$

$(f^{-1})'(1) = \frac{1}{3}$

28) $f(x) = x^5 + 2x^3 + x - 1$

(a) $f(1) = (1)^5 + 2(1)^3 + (1) - 1 = 3$

$(1,3) \rightarrow (3,1)$

$f'(x) = 5x^4 + 6x^2 + 1$

$f'(1) = 5(1)^4 + 6(1)^2 + 1$

$= 12$

(b) $f^{-1}(3) = 1$

$(f^{-1})'(3) = \frac{1}{12}$

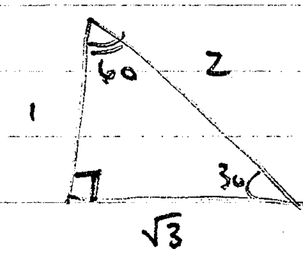
23) $y = \sec^{-1} x$, $x = 2$

$y' = \frac{1}{|x|\sqrt{x^2-1}}$

$y'(2) = \frac{1}{2\sqrt{4-1}} = \frac{1}{2\sqrt{3}} = m$

$y(2) = \sec^{-1}(2) = \frac{2}{1}$

$y(2) = \frac{\pi}{3}$



$$y - \frac{\pi}{3} = \frac{1}{2\sqrt{3}}(x-2)$$

3) $y = \sin^{-1} \sqrt{2}\, t$

$$y' = \frac{1}{\sqrt{1-(\sqrt{2}t)^2}}\left[\sqrt{2}\right]$$

$$= \frac{\sqrt{2}}{\sqrt{1-2t^2}}$$

3.8 $\qquad t^{-1}$

17) $y = \sec^{-1} \frac{1}{t}$

$$y' = \frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2 - 1}}\left[-\frac{1}{t^2}\right]$$

$$y' = \frac{-1}{\left|\frac{1}{t}\right|\sqrt{\frac{1}{t^2} - 1}} \cdot \frac{1}{t^2}$$

$$\frac{9}{3} \cdot \frac{8}{1} \qquad \frac{\frac{9}{1}}{\frac{3}{1}}$$

$$y' = \frac{-1}{\left|\frac{1}{t}\right|\sqrt{\frac{1-t^2}{t^2}}} \cdot \frac{1}{t^2}$$

$$= \frac{-1}{\left|\frac{1}{t}\right| \frac{\sqrt{1-t^2}}{\sqrt{t^2}}} \cdot \frac{1}{t^2}$$

$$= \frac{-1}{\left|\frac{1}{t}\right| \frac{\sqrt{1-t^2}}{|t|}} \cdot \frac{1}{t^2}$$

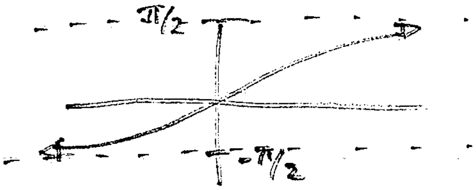
$$= \frac{-1}{\frac{\sqrt{1-t^2}}{\cancel{t^2}}} \cdot \frac{1}{\frac{\cancel{t^2}}{1}} = \frac{-1}{\sqrt{1-t^2}}$$

31) $x = \arctan t \qquad (t^2+1)^{-1}$

$v = \frac{1}{t^2+1} > 0$ for all t

$a = \frac{-(2t)}{(t^2+1)^2} < 0$ for $t > 0$

$\lim\limits_{t \to \infty} \arctan t$



3.8

5) $y = \sin^{-1}\left(\frac{3}{t^2}\right)$ $\qquad 3t^{-2} \qquad -6t^{-3}$

$$y' = \frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}}\left[\frac{-6}{t^3}\right]$$

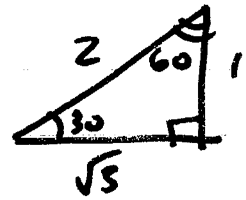
$$= \frac{1}{\sqrt{1-\frac{9}{t^4}}} \cdot \frac{-6}{t^3}$$

$$= \frac{1}{\sqrt{\frac{t^4-9}{t^4}}} \cdot \frac{-6}{t^3}$$

$$= \frac{1}{\frac{\sqrt{t^4-9}}{\cancel{t^2}}} \cdot \frac{-6}{\frac{\cancel{t^3}\,t}{1}} = \frac{-6}{t\sqrt{t^4-9}}$$

3.8

23) $y = \sec^{-1} x \quad , x = 2 \qquad (2, \frac{\pi}{6})$



$$y' = \frac{1}{|x|\sqrt{x^2-1}}$$

$$y'(2) = \frac{1}{2\sqrt{4-1}} = \frac{1}{2\sqrt{3}}$$

$$y = \frac{1}{2\sqrt{3}}(x-2) + \frac{\pi}{6}$$

$$y - \frac{\pi}{6} = \frac{1}{2\sqrt{3}}(x-2)$$

5) $y = \sin^{-1} \frac{3}{t^2} \qquad 3t^{-2}$

$$y' = \frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}}\left[-6t^{-3}\right]$$

$$y' = \frac{1}{\sqrt{1-\frac{9}{t^4}}} \cdot \frac{-6}{t^3}$$

$$y' = \frac{1}{\sqrt{\frac{1}{t^4}(t^4-9)}} \cdot \frac{-6}{t^3}$$

$$y' = \frac{1}{\sqrt{\frac{1}{t^4}}\sqrt{t^4-9}} \cdot \frac{-6}{t^3}$$

$$y' = \frac{1}{\frac{1}{t^2}\sqrt{t^4-9}} \cdot \frac{-6}{t^3}$$

$$y' = \frac{\cancel{t^2}}{\sqrt{t^4-9}} \cdot \frac{-6}{\cancel{t^3}\,t}$$

$$y' = \frac{-6}{t\sqrt{t^4-9}}$$

3.8

7) $y = x\sin^{-1}x + \sqrt{1-x^2}$

$y = \overset{f}{x}\sin^{-1}\overset{g}{x} + (1-x^2)^{1/2}$

$y' = \overset{f'g}{\sin^{-1}x} + \frac{\underbrace{x}_{fg'}}{\sqrt{1-x^2}} + \cancel{\tfrac{1}{2}}(1-x^2)^{-\frac{1}{2}}[-\cancel{2}x]$

$= \sin^{-1}x + \cancel{\frac{x}{\sqrt{1-x^2}}} - \cancel{\frac{x}{\sqrt{1-x^2}}}$

$= \boxed{\sin^{-1}x}$

$\frac{1}{4}t$

9) $x(t) = \sin^{-1}\left(\frac{t}{4}\right)$, $t = 3$

$x'(t) = \frac{1}{\sqrt{1-\left(\frac{t}{4}\right)^2}}\left[\frac{1}{4}\right]$

$= \frac{1}{\sqrt{1-\left(\frac{3}{4}\right)^2}}\left[\frac{1}{4}\right]$

$1 - \frac{9}{16}$

$\frac{16}{16} - \frac{9}{16} = \frac{7}{16}$

$= \frac{1}{\sqrt{\frac{7}{16}}} \cdot \frac{1}{4}$

$= \frac{1}{\frac{\sqrt{7}}{4}} \cdot \frac{1}{4}$

$= \frac{\cancel{4}}{\sqrt{7}} \cdot \frac{1}{\cancel{4}}$

$= \frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \boxed{\frac{\sqrt{7}}{7}}$

3.8

13) $y = \sec^{-1}(2s+1)$

$$y' = \frac{1}{|2s+1|\sqrt{(2s+1)^2-1}}\,[2]$$

$$y' = \frac{2}{|2s+1|\sqrt{(2s+1)^2-1}} = \frac{2}{|2s+1|\sqrt{4s^2+4s+\cancel{1}-\cancel{1}}}$$

no!

$$= \frac{2}{|2s+1|\sqrt{4(s^2+s)}}$$

$$= \frac{\cancel{2}}{|2s+1|\cdot\cancel{2}\cdot\sqrt{s^2+s}}$$

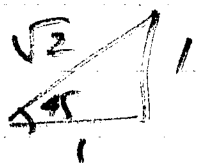
$$= \frac{1}{|2s+1|\sqrt{s^2+s}}$$

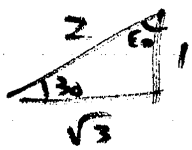
foil
FOIL

15) $y = \csc^{-1}(x^2+1)$

$$y' = \frac{-1}{|x^2+1|\sqrt{(x^2+1)^2-1}}\,[2x]$$

$$= \frac{-2x}{|x^2+1|\sqrt{(x^2+1)^2-1}}$$





25) $y = \sin^{-1}\left(\frac{x}{4}\right)$ $\frac{d}{dx}$, $x = 3$ $\boxed{\left(3, \sin^{-1}\frac{3}{4}\right)}$

$$y' = \frac{1}{\sqrt{1-\left(\frac{x}{4}\right)^2}}\left[\frac{1}{4}\right]$$ $$y - \sin^{-1}\frac{3}{4} = \frac{1}{\sqrt{7}}(x-3)$$

$$y'(3) = \frac{1}{\sqrt{1-\left(\frac{3}{4}\right)^2}}\left[\frac{1}{4}\right]$$

$$= \frac{1}{\sqrt{\frac{7}{16}}}\cdot\frac{1}{4} \qquad \frac{\cancel{4}}{\sqrt{7}}\cdot\frac{1}{\cancel{4}} = \boxed{\frac{1}{\sqrt{7}} = m}$$

3.8

22) $y = \cot^{-1}\frac{1}{x} - \tan^{-1} x$ $\quad$ (x^{-1})

$$y' = \frac{-1}{1+\left(\frac{1}{x}\right)^2}\left[-x^{-2}\right] - \frac{1}{1+x^2}$$

$$y' = \frac{-1}{\left(1+\frac{1}{x^2}\right)} \cdot \frac{-1}{x^2} - \frac{1}{1+x^2}$$

$$= \frac{1}{x^2+1} - \frac{1}{1+x^2} = 0$$

3.8

15) $y = \csc^{-1}(x^2+1)$

$$y' = \frac{-1}{|x^2+1|\sqrt{(x^2+1)^2-1}}\,[2x]$$

$$y' = \frac{-2x}{(x^2+1)\sqrt{x^4+2x^2+\cancel{1}-\cancel{1}}}$$

$$= \frac{-2x}{(x^2+1)\sqrt{x^2(x^2+2)}}$$

$$= \frac{-2\cancel{x}}{(x^2+1)\cancel{|x|}\sqrt{x^2+2}}$$

$$= \frac{-2}{(x^2+1)\sqrt{x^2+2}}$$

$\to (1-x^2)^{1/2}$ $\quad \frac{1}{(1-x^2)^{\frac{1}{2}}}$

7) $y = \overset{f}{x}\,\overset{g}{\sin^{-1}x} + \sqrt{1-x^2}$

$$y' = \sin^{-1}x + \overset{g'}{\frac{1}{\sqrt{1-x^2}}}\overset{f}{(x)} + \frac{1}{2}(1-x^2)^{-\frac{1}{2}}\,\boxed{-2x}$$

$$= \sin^{-1}x + \cancel{\frac{x}{\sqrt{1-x^2}}} - \cancel{\frac{x}{\sqrt{1-x^2}}}$$

$$\boxed{y' = \sin^{-1}x}$$

29) $f(x) = \cos x + 3x$ $\quad f'(x) = -\sin x + 3$

$f(0) = \cos(0)+3(0) = 1$ $\quad f'(0) = -\sin(0)+3 = 3$

$(0,1) \to (1,0)$

$(f^{-1})(1) = 0$ $\quad (f^{-1})'(1) = \frac{1}{3}$

$\int$

$f^{-1}(x)$