3.9

29) $y = 3^x + 1$

$y' = \dfrac{3^x \cancel{\ln 3}}{\cancel{\ln 3}} = \dfrac{5}{\ln 3}$

$y = 5x - 1$

$m = 5$

$\ln 3^x = \ln\left(\dfrac{5}{\ln 3}\right)$

$\dfrac{x(\ln 3)}{\ln 3} = \dfrac{\ln\left(\frac{5}{\ln 3}\right)}{\ln 3} = x$

31) $\boxed{y = mx + b}$

$\dfrac{y}{x} = \dfrac{mx}{x}$

$\boxed{\dfrac{y}{x} = m}$

$\dfrac{1}{x} = m$

$\dfrac{1}{\frac{e}{2}} = m$

$\boxed{\dfrac{2}{e} = m}$

$y = \ln(2x)$

$y' = \dfrac{1}{2x}[2] = \boxed{\dfrac{1}{x} = m}$

$\cancel{x} \cdot \dfrac{y}{\cancel{x}} = \dfrac{1}{\cancel{x}} \cdot \cancel{x}$

$y = 1$

$y = 1$

$1 = \ln(2x)$

$e^1 = e^{\ln(2x)}$

$\dfrac{e^1}{2} = \dfrac{2x}{2}$

$\dfrac{e}{2} = x$

3.9

45) $y = \sqrt[5]{\dfrac{(x-3)^4(x^2+1)}{(2x+5)^3}}$ stuff $= \dfrac{(x-3)^4(x^2+1)}{(2x+5)^3}$

$\ln y = \ln(\text{stuff})^{1/5}$

$\ln y = \frac{1}{5}\ln\left(\dfrac{(x-3)^4(x^2+1)}{(2x+5)^3}\right)$

$\ln y = \frac{1}{5}\left[\ln(x-3)^4 + \ln(x^2+1) - \ln(2x+5)^3\right]$

$\ln y = \frac{1}{5}\left[4\ln(x-3) + \ln(x^2+1) - 3\ln(2x+5)\right]$

$\ln y = \frac{4}{5}\ln(x-3) + \frac{1}{5}\ln(x^2+1) - \frac{3}{5}\ln(2x+5)$

$\frac{1}{y}\frac{dy}{dx} = \frac{4}{5}\cdot\frac{1}{x-3} + \frac{1}{5}\,\frac{1}{x^2+1}[2x] - \frac{3}{5}\,\frac{1}{2x+5}[2]$

$\cancel{y}\cdot\cancel{\frac{1}{y}}\frac{dy}{dx} = \frac{4}{5x-15} + \frac{2x}{5x^2+5} - \frac{6}{10x+25}\cdot y$

$= \left(\quad\downarrow \quad\downarrow \quad\downarrow\quad\right)\sqrt[5]{\text{stuff}}$

51) $P(t) = \dfrac{300}{1+2^{4-t}} = 300\cdot(1+2^{4-t})^{-1}$

(a) $\dfrac{300}{1+16} = 18$ish

(b) $P'(t) = -300\,(1+2^{4-t})^{-2}\left[2^{4-t}\ln 2\,[-1]\right]$

$= +300\cdot\frac{1}{4}\cdot\ln 2 = 75\ln 2$

3.9

45) $y = \sqrt[5]{\dfrac{(x-3)^4(x^2+1)}{(2x+5)^3}}$ stuff $= \dfrac{(x-3)^4(x^2+1)}{(2x+5)^3}$

$\ln y = \ln\left(\dfrac{(x-3)^4(x^2+1)}{(2x+5)^3}\right)^{\frac{1}{5}}$

$\ln y = \frac{1}{5}\ln\dfrac{(x-3)^4(x^2+1)}{(2x+5)^3}$

$\ln y = \frac{1}{5}\left[\ln (x-3)^4(x^2+1) - \ln (2x+5)^3\right]$

$\ln y = \frac{1}{5}\left[\ln (x-3)^4 + \ln (x^2+1) - \ln (2x+5)^3\right]$

$\ln y = \frac{1}{5}\left[4\ln (x-3) + \ln (x^2+1) - 3\ln (2x+5)\right]$

$y \cdot \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{5}\left[\dfrac{4}{x-3} + \dfrac{1}{x^2+1}[2x] - \dfrac{3}{2x+5}[2]\right] y$

$= \left[\dfrac{4}{5(x-3)} + \dfrac{2x}{5(x^2+1)} - \dfrac{6}{5(2x+5)}\right]\sqrt[5]{\text{stuff}}$

47) $y = x^{\ln x}$

$\ln y = \ln x^{\ln x}$

$\ln y = \ln x \cdot \ln x$

$\ln y = (\ln x)^2$

$y \cdot \dfrac{1}{y}\dfrac{dy}{dx} = \left(2(\ln x)\left[\dfrac{1}{x}\right]\right) y$

$= \dfrac{2\ln x}{x} \cdot x^{\ln x}$

$$\overset{\downarrow}{f} \cdot \overset{\downarrow}{g}$$

$$c \cdot g \qquad c g'$$

3.9

25) $y = \ln 2 \cdot \log_2 x$

$$\frac{dy}{dx} = \cancel{\ln 2} \frac{1}{x \cancel{\ln 2}} = \frac{1}{x}$$

29) $y = 3^x + 1$

$y = 5x - 1 \leftarrow \parallel$

$m = 5$

$$y' = \frac{3^x \cancel{\ln 3}}{\cancel{\ln 3}} = \frac{5}{\ln 3}$$

$$\ln 3^x = \ln\left[\frac{5}{\ln 3}\right]$$

$$\frac{x \cancel{\ln 3}}{\cancel{\ln 3}} = \frac{\ln\left[\frac{5}{\ln 3}\right]}{\ln 3}$$

43) $y = (\sin x)^x$

$$\ln y = \ln(\underset{f}{\sin} \underset{g}{x})^x$$

$$\ln y = x \ln(\sin x)$$

$$y \cdot \frac{1}{y} \frac{dy}{dx} = \left(\ln(\sin x) + \frac{1}{\sin x}[\cos x]\, x\right) y$$

$$= \left(\ln(\sin x) + x \cot x\right) (\sin x)^x$$

23) $y = \log_2\left(\frac{1}{x}\right) = \log_2(x^{-1})$

$$\frac{dy}{dx} = \frac{x}{\ln 2}\left[-x^{-2}\right]$$

$$= \frac{\cancel{x}}{\ln 2}\left[-\frac{1}{x^{\cancel{2}}}\right]$$

$$= \boxed{\frac{-1}{x \ln 2}}$$

3.9

9) $y = e^{\sqrt{x}}$

$y = e^{x^{1/2}}$

$\frac{dy}{dx} = e^{x^{1/2}} \left[\frac{1}{2}x^{-1/2}\right]$

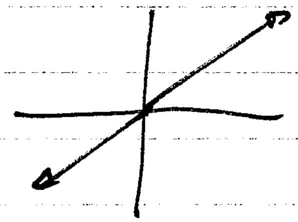
$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

7) $y = xe^2 - e^x$

$\frac{dy}{dx} = e^2 - e^x$

31) $y = \ln(2x)$

$\frac{dy}{dx} = \frac{1}{\cancel{2}x}[\cancel{2}] = \frac{1}{x} = m$



$\frac{y}{x} = \frac{mx}{x}$

$\frac{y}{x} = m$

$\frac{1}{x} = \frac{y}{x}$

$1 = y$

$\boxed{\left(\frac{e}{2}, 1\right)}$

$\cancel{1 = \ln(2x)}$

$\cancel{e = 2x}$

$1 = \ln(2x)$

$e^1 = e^{\ln(2x)}$

$\frac{e}{2} = \frac{2x}{2}$

17) $y = \ln\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = \cancel{x}\left[-\frac{1}{\cancel{x^2}}\right] = \boxed{-\frac{1}{x}}$

$y = \ln(x^{-1})$

$y = -\ln x$

$\boxed{\frac{dy}{dx} = -\frac{1}{x}}$

$\cancel{[\ln(x)]^{-1}}$

$\cancel{\frac{-[\ln x]^{-2}}{x}}$

3.9

30) $y = 2e^x - 1$

$y = -3x + 2$

$m = -3$

$\frac{1}{3}$

$y' = \frac{\cancel{2}e^x}{\cancel{2}} = \frac{1}{3} \cdot \frac{1}{2}$

$\ln e^x = \ln \frac{1}{6}$

$x = \ln \frac{1}{6}$

44) $y = x^{\tan x}$

$\ln y = \ln x^{\tan x}$

$\ln y = (\tan x)(\ln x)$

$y \cdot \frac{1}{y}\frac{dy}{dx} = [(\sec^2 x)(\ln x) + (\frac{1}{x})(\tan x)]\,y$

$\frac{dy}{dx} = \left[(\sec^2 x)(\ln x) + \frac{\tan x}{x}\right] x^{\tan x}$

50) $y = xe^x$

$y' = e^x + e^x x$

$y'(a) = e^a + e^a(a)$ TANGENT

~~TANGENT~~ NORMAL

$(a, ae^a), (0,0)$

$\frac{ae^a - 0}{a - 0} = \frac{ae^a}{a} = e^a$

$(e^a + ae^a) \quad \frac{-1}{\cancel{e^a + ae^a}} = e^a(\cancel{a}e^a + ae^a)$

$-1 = e^a(e^a + ae^a)$

$-1 = e^{2a} + ae^{2a}$

$+1 \qquad\qquad +1$

$0 = e^{2a} + ae^{2a} + 1$

3.9

51) $P(t) = \frac{300}{1+2^{4-t}}$

$$17\overline{)300}$$ 18 r 5
17
130
136

(a) $P(0) = \frac{300}{1+2^4} \approx 18$ students

(b) $P(t) = 300(1+2^{4-t})^{-1}$

$P'(t) = -300(1+2^{4-t})^{-2}\left[2^{4-t}(\ln 2)[-1]\right]$

$P'(4) = -300(1+2^{4-4})^{-2}\left[2^{4-4}(\ln 2)[-]\right]$

$= +300(2)^{-2}[\ln 2]$

$= \frac{300}{4}\ln 2$

$= 75\ln 2$