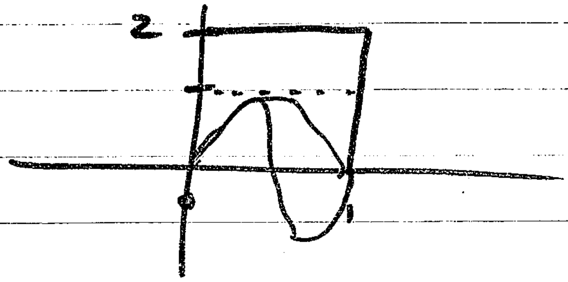
5.3

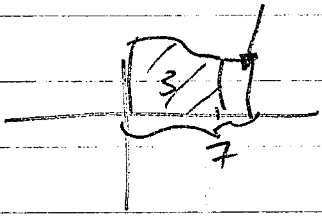
7) $\int_0^1 \sin(x^2) \neq 2$

max = 1

min = -1



5) $\int_0^3 f(z)\,dz = 3 \quad \int_0^4 f(z)\,dz = 7$



(a) $\int_3^4 f(z)\,dz = \int_0^4 f(z)\,dz - \int_0^3 f(z)\,dz$

$7 - 3 = 4$

$\int_0^3 f(z)\,dz + \int_3^4 f(z)\,dz = \int_0^4 f(z)\,dz$

$-\int_0^3 \qquad\qquad -\int_0^3$

1) (d) $\int_2^5 f(x)\,dx = \int_1^5 - \int_1^2$

$6 - -4 = 10$

5.3

40) (a) 300 mi

(b) $\frac{150}{30} = 5 \qquad \frac{150}{50} = 3$

8 hours

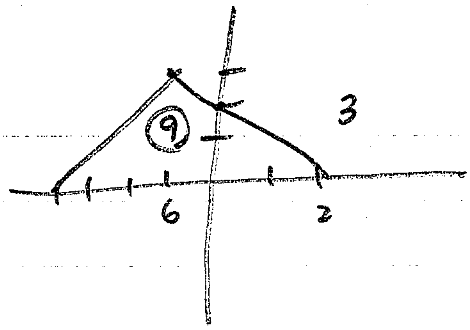
$$\frac{\frac{150}{30} + \frac{150}{5}}{2}$$

(c) $\frac{300}{8} = 37.5$ mph

(d) $\frac{d_1 + d_2}{t_1 + t_2}$

$$\frac{1}{2}\left(\frac{d_1}{t_1} + \frac{d_2}{t_2}\right)$$

15) $f(x) = \begin{cases} x+4, & -4 \le x \le -1 \\ -x+2, & -1 < x \le 2 \end{cases}$

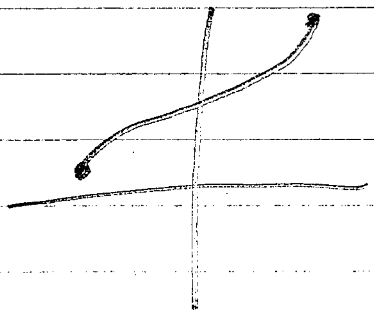


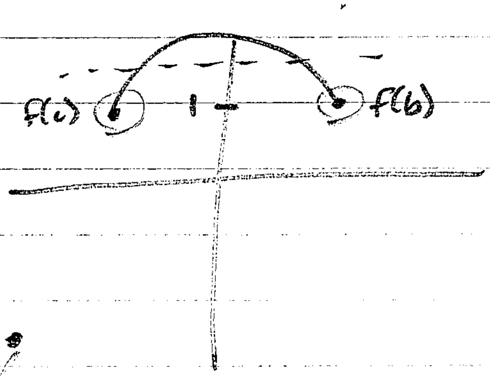
$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

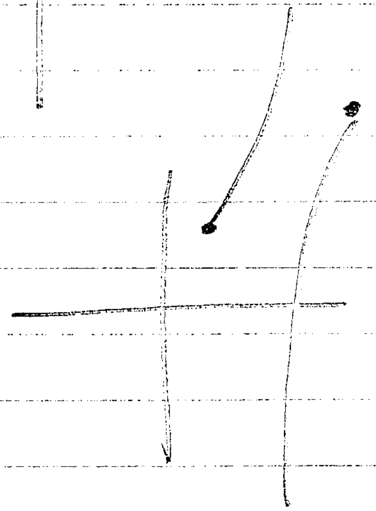
$$\frac{1}{2--4}\underbrace{\int_{-4}^{2} f(x)\,dx}$$

$$\frac{1}{6}\,9 = \frac{3}{2}$$

45)

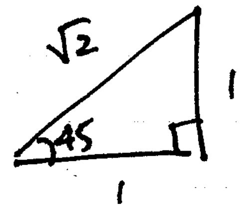


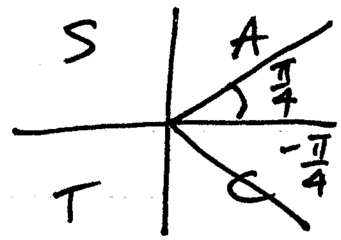




5.3

27) $\int_{-1}^{1} \frac{1}{1+x^2}\,dx$





$\Big|_{-1}^{1} \tan^{-1} x$

$\tan^{-1}(1) - \tan(-1)$

$\frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

5) $\int_0^3 f(z)\,dz = 3$ $\qquad \int_0^4 f(z)\,dz = 7$

(a) $\int_3^4 f(z)\,dz = 4$

$$\int_0^4 f(z)\,dz = \int_0^3 f(z)\,dz + \int_3^4 f(z)\,dz$$

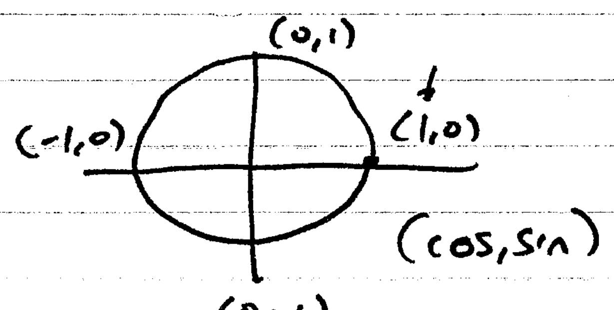
$$7 = 3 + \int_3^4 f(z)\,dz$$

(b) $\int_4^3 f(t)\,dt = -4$

$\int_0^{10} x^3\,dx$ $\qquad \int_0^{10} t^3\,dt$ $\qquad \underline{t = 3z^{12}}$

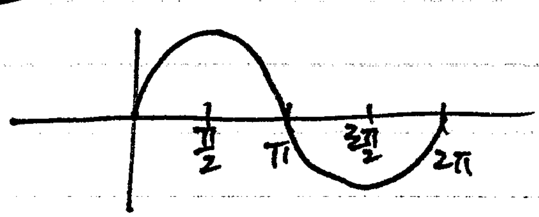
19) $\int_{\pi}^{2\pi} \sin x\,dx$

$\Big|_{\pi}^{2\pi} -\cos x$



$-\cos(2\pi) - -\cos(\pi)$

$-(1) + (-1) = -2$



5.3

21) $\int_0^1 e^x \, dx$

$e^x \Big|_0^1$

$e^1 - e^0$

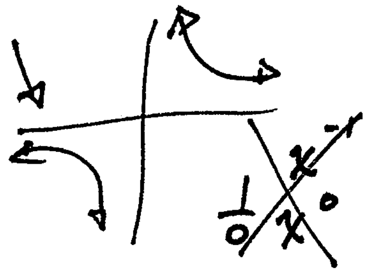
$e - 1$

29) $\int_1^e \frac{1}{x} \, dx$

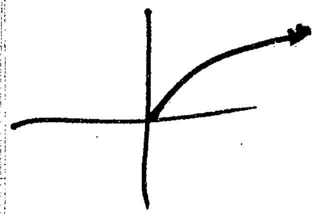
$\ln|x| \Big|_1^e$

$\ln e - \ln 1$

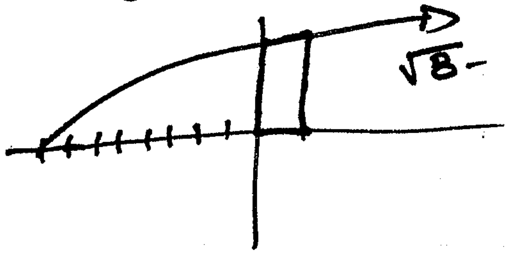
$1 - 0 = 1$

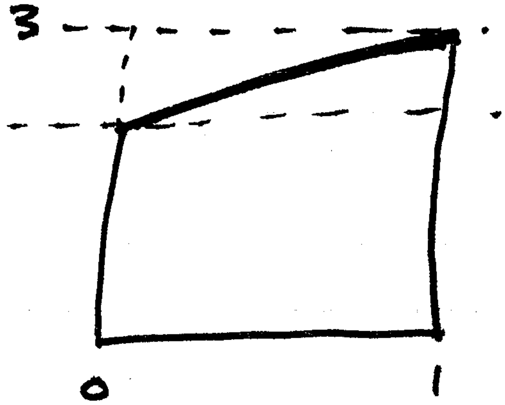


8) $y = \sqrt{x}$



$y = \sqrt{x+8}$





$Area_{MAX} = 1 \times 3$

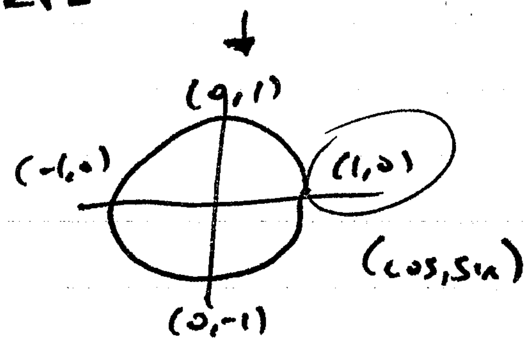
$Area_{MIN} = 1 \times \sqrt{8} = \sqrt{8} \rightarrow 2\sqrt{2}$

20) $\int_0^{\pi/2} \cos x \, dx$

$\sin x \Big|_0^{\pi/2}$

$\sin\left(\frac{\pi}{2}\right) - \sin(0)$

$1 - 0 = 1$



23) $\int_1^4 2x \, dx$

$x^2 \Big|_1^4$

$(4)^2 - (1)^2$

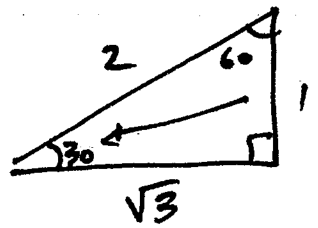
$16 - 1 = 15$

28) $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}} \, dx$

$\sin^{-1} x \Big|_0^{1/2}$

$\sin^{-1}\left(\frac{1}{2}\right) - \sin(0)$

$\frac{\pi}{6} - 0 = \frac{\pi}{6}$



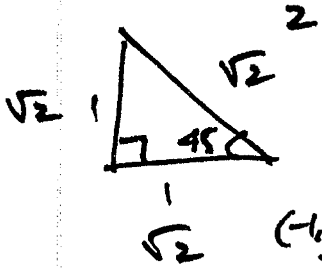
$\frac{1}{2} \quad \frac{opp}{hyp}$

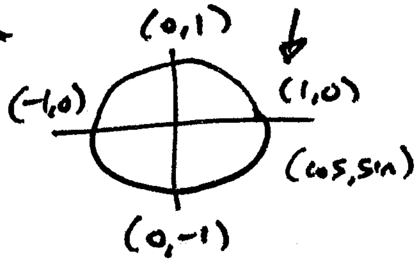
$\cancel{30} \cdot \frac{\pi}{\cancel{180}}$ 6

5.3

33) $y = \sec^2 x \;,\; [0, \frac{\pi}{4}]$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$





$$= \frac{1}{\frac{\pi}{4}-0}\int_0^{\pi/4} \sec^2 x\,dx$$

$$= \frac{4}{\pi}\Big|_0^{\pi/4} \tan x$$

$$= \frac{4}{\pi}\left[\tan\frac{\pi}{4} - \tan 0\right]$$

$$= \frac{4}{\pi}[1-0]$$

$$= \frac{4}{\pi}$$

$\cancel{45}\cdot\frac{\pi}{\cancel{180}}$ $\quad 4$

32) $y = \frac{1}{x} \;,\; [e, 2e]$

$\ln(ab) = \ln a + \ln b$

$$f(c) = \frac{1}{2e-e}\int_e^{2e} \frac{1}{x}\,dx$$

$$= \frac{1}{e}\Big|_e^{2e} \ln|x|$$

$$= \frac{1}{e}[\ln 2e - \ln e]$$

$$= \frac{1}{e}[\ln 2 + \ln e - \ln e]$$

$$= \frac{\ln 2}{e}$$

13) $y = -3x^2 - 1 \;,\; [0,1]$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{1-0}\int_0^1 (-3x^2-1)\,dx$$

$$= \Big|_0^1 -x^3 - x$$

$$[(-(1)^3 - (1)) - (\cancel{-(0)^3 - 0})]$$

-2

$$-3x^2 - 1 = -2$$

$$-3x^2 = -1$$

$$x^2 = \frac{1}{3}$$

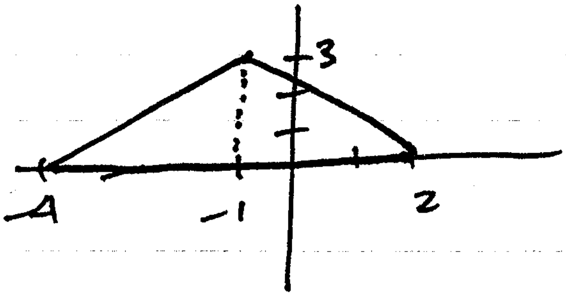
$$x = \pm\sqrt{\frac{1}{3}}$$

$$c = \sqrt{\frac{1}{3}}$$

$$c = \frac{1}{\sqrt{3}}, \frac{\sqrt{3}}{3}$$

5.3

15) $f(x) = \begin{cases} x+4 \\ -x+2 \end{cases}$



$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

$= \frac{1}{2--4}\int_{-4}^{2} f(x)\,dx$

$= \frac{1}{6}\int_{-4}^{2} f(x)\,dx$

$= \frac{1}{6}(9) = \frac{3}{2}$

$\frac{1}{2}(6)(3) = 9$

40) (a) 300 miles

(b) $\frac{150}{30} = 5$ hours $\quad \frac{150}{50} = 3$ hours $\quad 5+3=8$ hours

(c) $\frac{300}{8} = 37.5$ mph

50 TEST - DERIVATIVES → 50%

80 TEST - INTEGRALS → 70%

→ 90%

$130/2 = 65\%$ → 80%

72.5

41) TOTAL AMOUNT OF WATER: $2000\ m^3$

TOTAL AMOUNT OF TIME: $\frac{1000}{10} = 100$ MIN $\quad \frac{1000}{20} = 50$ MIN $\quad$ 50 MIN

AVERAGE RATE: $\frac{2000}{150} = \boxed{13\tfrac{1}{3}\ m^3/\text{MIN}}$

5.3

11) $y = x^2 - 1$, $[0, \sqrt{3}]$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{\sqrt{3}-0}\int_0^{\sqrt{3}} (x^2-1)\,dx$$

$$= \frac{1}{\sqrt{3}} \Big|_0^{\sqrt{3}} \frac{1}{3}x^3 - x$$

$$\frac{1}{\sqrt{3}}\left[\left(\frac{1}{3}(\sqrt{3})^3 - \sqrt{3}\right) - \cancel{\left(\frac{1}{3}(0)^3 - 0\right)}\right]$$

$$\frac{1}{\sqrt{3}}\left[\cancel{\frac{1}{3}}(\cancel{3}\sqrt{3}) - \sqrt{3}\right]$$

$$\frac{1}{\sqrt{3}}\left[\sqrt{3} - \sqrt{3}\right]$$

$$f(c) = \frac{1}{\sqrt{3}} \cdot 0 = 0$$

$$\sqrt{27}$$
$$\sqrt{9 \cdot 3}$$
$$\sqrt{9}\sqrt{3}$$
$$3\sqrt{3} \quad (\sqrt{3})^3$$
$$(\sqrt{3})^2(\sqrt{3})$$
$$3\sqrt{3}$$

$$x^2 - 1 = 0$$
$$x^2 = 1$$
$$x = \pm 1$$
$$c = 1$$

35) $y = 3x^2 + 2x$, $[-1, 2]$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{2--1}\int_{-1}^{2} (3x^2+2x)\,dx$$

$$= \frac{1}{3} \Big|_{-1}^{2} x^3 + x^2$$

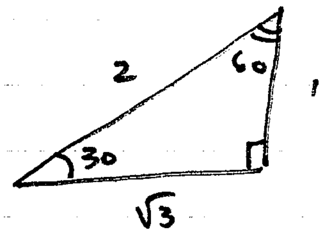
$$= \frac{1}{3}\left[(2^3+2^2) - (\cancel{(-1)^3 + (-1)^2})\right]$$

$$= \frac{1}{3}[12] = 4$$

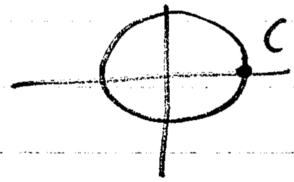
5.3

$$\frac{180°}{3} = 60$$

36) $y = \sec x \tan x \quad , \quad [0, \frac{\pi}{3}]$



$$\frac{1}{\frac{\pi}{3} - 0} \int_0^{\pi/3} \sec x \tan x \, dx$$

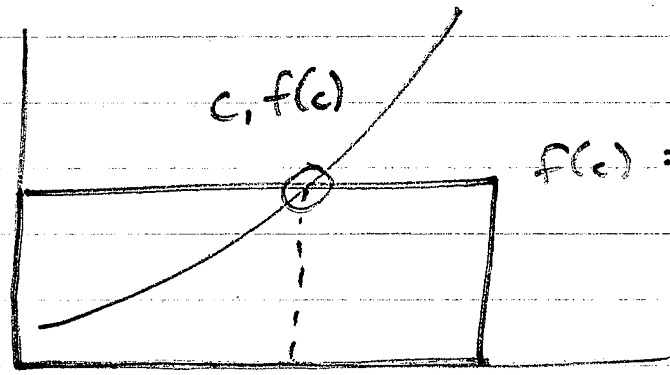


$$\frac{3}{\pi} \Big|_0^{\pi/3} \sec x$$

$$\frac{3}{\pi} \left[\sec \frac{\pi}{3} - \sec 0\right]$$

$\frac{3}{\pi}[2 - 1] = \frac{3}{\pi} \rightarrow$ AVERAGE VALUE, $f(c)$

$$\frac{3}{\pi} = \sec x \tan x$$



$f(c) = \frac{3}{\pi}$

$C = .64999$