6.2

$\int \tan x \, dx$

$-\int \frac{\sin x}{\cos x} \underbrace{dx\,[-]}_{du}$ $\quad u = \cos x$

$du = -\sin x \, dx$

$-\int \frac{1}{u} \, du$

$-\ln|u| + C$

$\boxed{-\ln|\cos x| + C}$

$\ln|\cos x|^{-1} + C$

$\boxed{\ln|\sec x| + C}$

17) $\int \sin^3 2x \, dx$ $\qquad \sin^2 2x = 1 - \cos^2 2x$

$\int \sin 2x (\sin^2 2x) \, dx$

$\int \sin 2x (1 - \cos^2 2x) \, dx$

$\frac{1}{2}\int \sin 2x \, dx^{[2]} - \frac{1}{2}\int \cos^2 2x \sin 2x \, dx^{[2]}$ du

$u = 2x$

$du = 2\,dx$ $\qquad d\mathcal{B}$

$\frac{1}{2}\int \sin u \, du + \frac{1}{2}\int \cos^2 u \sin u \, du$

$-\frac{1}{2}\cos u$ $\qquad \mathcal{B} = \cos u$

$d\mathcal{B} = -\sin u \, du$

$-\frac{1}{2}\cos u + \frac{1}{2}\int \mathcal{B}^2 \, d\mathcal{B}$

$-\frac{1}{2}\cos u + \frac{1}{2}\cdot\frac{1}{3}\mathcal{B}^3 + C$

$-\frac{1}{2}\cos u + \frac{1}{6}\cos^3 u + C$

$-\frac{1}{2}\cos 2x + \frac{1}{6}\cos^3 2x + C$

6.2

49) $\int 2\sin^2 x \, dx$ $\qquad \cos 2x = \boxed{2\sin^2 x} - 1$

$+1 \qquad\qquad +1$

$\int (\cos 2x + 1)\, dx$

$\frac{1}{2}\int \cos 2x \, \underbrace{dx[2]}_{du} + \int 1 \, dx$

$u = 2x$

$du = 2dx$

$\frac{1}{2}\int \cos u \, du$

$\frac{1}{2}\sin u$

$\frac{1}{2}\sin 2x + x + C$

51) $\int \tan^4 x \, dx$ $\qquad \tan^2 x = \sec^2 x - 1$

$\int \tan^2 x \tan^2 x \, dx$

$\int \tan^2 x (\sec^2 x - 1)\, dx$

$\int \tan^2 x \sec^2 x \, dx - \int \tan^2 x \, dx$

$\int \tan^2 x \, \overset{du}{\sec^2 x \, dx} - \int (\sec^2 x - 1)\, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$\int u^2 \, du$

$\frac{1}{3}u^3$

$\boxed{\frac{1}{3}\tan^3 x - \tan x + x + C}$

69) $y = \ln\left|\frac{\cos 3}{\cos x}\right| + 5$ $\qquad \frac{dy}{dx} = \tan x, \; f(3) = 5$ ✓

$y(3) = \ln\left|\frac{\cos 3}{\cos 3}\right| + 5 = 5$ $\qquad y = \ln|\cos 3| - \ln|\cos x| + 5$

$y' = \frac{-1}{\cos x}[-\sin x]$

$y' = \frac{\sin x}{\cos x} = \tan x$ ✓

6.2

51) $\int \tan^4 x \, dx$

$\int \tan^2 x \tan^2 x \, dx$

$\int \tan^2 x (\sec^2 x - 1) \, dx$

$\int \tan^2 x \sec^2 x \, dx - \int \tan^2 x \, dx$

$\int \tan^2 x \underbrace{\sec^2 x \, dx}_{du} - \int (\sec^2 x - 1) \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$\int u^2 \, du$

$\frac{1}{3} u^3$

$\frac{1}{3} \tan^3 x - \tan x + x + C$

49) $\int 2 \sin^2 x \, dx$

$\cos 2x = 2\sin^2 x - 1$

$\cos 2x + 1 = 2\sin^2 x$

$\int (\cos 2x + 1) \, dx$

$\frac{1}{2} \int \cos 2x \underbrace{dx}_{du} [2] + \int 1 \, dx$

$u = 2x$

$du = 2 \, dx$

$\frac{1}{2} \int \cos u \, du$

$\frac{1}{2} \sin u$

$\frac{1}{2} \sin 2x + x + C$

47) $\int \sin^3 2x \, dx$

$\sin^2 2x = 1 - \cos^2 2x$

$\int \sin 2x \sin^2 2x \, dx$

$\int \sin 2x (1 - \cos^2 2x) \, dx$

$\frac{1}{2} \int \sin 2x \underbrace{dx}_{du} 2 - \left(-\frac{1}{2}\right) \int \cos^2 2x \sin 2x \, dx \quad [-2]$

$u = 2x \qquad u = \cos 2x$

$du = 2 \qquad du = -\sin 2x \, [2] \, dx$

$\frac{1}{2} \int \sin u \, du + \frac{1}{2} \int u^2 \, du$

$-\frac{1}{2} \cos u + \frac{1}{2} \cdot \frac{1}{3} u^3 + C$

$-\frac{1}{2} \cos 2x + \frac{1}{6} \cos^3 2x + C$

6.2

9) $\frac{d}{dx}\ \frac{1}{2}e^{2x}+C$

$\cancel{\frac{1}{2}}e^{2x}\ [\cancel{2}]$

e^{2x}

13) $f(u)=\sqrt{u} \qquad u=x^2$

$\int f(u)\,du = \int \sqrt{u}\,du = \int u^{1/2}\,du = \frac{2}{3}u^{3/2}+C = \frac{2}{3}(x^2)^{3/2}+C$

$\boxed{\frac{2}{3}x^3+C}$

$\int f(u)\,dx = \int \sqrt{u}\,dx = \int \sqrt{x^2}\,dx = \int x\,dx = \boxed{\frac{1}{2}x^2+C}$

15) $f(u)=e^u \qquad u=7x$

$\int f(u)\,du = \int e^u\,du = e^u+C = \boxed{e^{7x}+C}$

$\int f(u)\,dx = \frac{1}{7}\int e^u \underbrace{dx\,[7]}_{du} = \frac{1}{7}\int e^u\,du = \frac{1}{7}e^u+C = \boxed{\frac{1}{7}e^{7x}+C} \qquad u=7x$

$du = 7\,dx$

21) $\int \frac{dx}{x^2+9} \qquad u=\frac{x}{3} \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a}+C$

$\int \frac{3\,du}{(3u)^2+9} \qquad 3u = x$

$3\int \frac{1}{9u^2+9}\,du \qquad 3du = dx \qquad u^2=x^2 \quad a^2=9$

$3\int \frac{1}{9(u^2+1)}\,du \qquad u=x \quad a=3$

$\frac{3}{9}\int \frac{1}{u^2+1}\,du \qquad du=dx$

$\frac{1}{3}\tan^{-1}u+C$

$\frac{1}{3}\tan^{-1}\frac{x}{3}+C \longleftarrow \boxed{\frac{1}{3}\arctan\frac{x}{3}+C}$

6.2

67) (a) $\int_0^1 \frac{x^3}{\sqrt{x^4+9}}\,dx = \frac{1}{4}\int_0^1 (x^4+9)^{-1/2}\, x^3\,dx\,[4]\; du$

$u = x^4+9$

$du = 4x^3\,dx$

$\frac{1}{4}\int_9^{10} u^{-1/2}\,du$

$\frac{1}{4}\Big|_9^{10} 2u^{1/2}$

$\frac{1}{4}\left[2\sqrt{10} - 2\sqrt{9}\right]$

$\boxed{\frac{1}{4}\left[2\sqrt{10} - 6\right]}$

(b) $\frac{1}{4}\int_0^1 (x^4+9)^{-1/2}\, x^3\,dx\; 4\; du$

$\frac{1}{4}\int u^{-1/2}\,du$

$\frac{1}{4}\int 2u^{1/2}$

$\frac{1}{4}\Big|_0^1 2(x^4+9)^{1/2}$

$\frac{1}{4}\left[(2(1^4+9))^{1/2} - (2(0^4+9))^{1/2}\right]$

$\boxed{\frac{1}{4}\left[2\sqrt{10} - 6\right]}$