6.3

53) $\int \sin^{-1}x\,dx \quad = x\sin^{-1}x - \int \frac{1}{\sqrt{1-x^2}}\,x\,dx$

$u = \sin^{-1}x \qquad v = x \qquad x\sin^{-1}x - \frac{1}{-2}\int (1-x^2)^{-1/2}\,x\,dx\,[-2]\ du$

$du = \frac{1}{\sqrt{1-x^2}}dx \quad dv = dx \qquad u = 1-x^2$

$du = -2x\,dx$

$x\sin^{-1}x + \frac{1}{2}\int (u)^{-1/2}\,du$

$+\frac{1}{2}\cdot 2u^{1/2} + C$

$\boxed{x\sin^{-1}x + \sqrt{1-x^2} + C}$

56) $\int \log_2 x\,dx \quad = x\log_2 x - \int \frac{1}{\cancel{x}(\ln 2)}\cdot\cancel{x}\,dx$

$u = \log_2 x \qquad v = x$

$du = \frac{1}{x(\ln 2)}dx \quad dv = dx \qquad x\log_2 x - \int \frac{1}{\ln 2}\,dx$

$\boxed{x\log_2 x - \frac{1}{\ln 2}x + C}$

31) $\frac{dy}{d\theta} = \theta\sec^{-1}\theta \quad = \frac{1}{2}\theta^2\sec^{-1}\theta - \int \frac{1}{2}\cancel{\theta^2}^{\theta}\frac{1}{\cancel{|\theta|}\sqrt{\theta^2-1}}\,d\theta$

$\int dy = \int \theta\sec^{-1}\theta\,d\theta \qquad \frac{1}{2}\theta^2\sec^{-1}\theta - \frac{1}{2}\int \frac{1}{\sqrt{\theta^2-1}}\,\theta\,d\theta$

$y = \quad u = \sec^{-1}\theta \quad v = \frac{1}{2}\theta^2 \qquad \frac{1}{2}\theta^2\sec^{-1}\theta - \frac{1}{2}\frac{1}{2}\int(\theta^2-1)^{-1/2}\,\theta\,d\theta\,2$

$du = \frac{1}{|\theta|\sqrt{\theta^2-1}}d\theta \quad dv = \theta\,d\theta \qquad u = \theta^2 - 1$

$du = 2\theta\,d\theta$

$\frac{1}{2}\theta^2\sec^{-1}\theta - \frac{1}{4}\int u^{-1/2}\,du$

$\frac{1}{2}\theta^2\sec^{-1}\theta - \frac{1}{4}\,2u^{1/2} + C$

$\boxed{\frac{1}{2}\theta^2\sec^{-1}\theta - \frac{1}{2}\sqrt{\theta^2-1} + C}$

6.3

20 $\int e^{-x} \sin 2x \, dx = -e^{-x} \sin 2x - \int -2e^{-x} \cos 2x \, dx$

$u = \sin 2x \quad v = -e^{-x}$

$du = 2\cos 2x \, dx \quad dv = e^{-x} dx$

$= -e^{-x} \sin 2x + 2\int e^{-x} \cos 2x \, dx$

$u = \cos 2x \quad v = -e^{-x}$

$du = -2\sin 2x \, dx \quad dv = e^{-x} dx$

$= -e^{-x} \sin 2x + 2\left[-e^{-x} \cos 2x - \int 2e^{-x} \sin 2x \, dx\right]$

$\int e^{-x} \sin 2x \, dx = -e^{-x} \sin 2x - 2e^{-x} \cos 2x - \cancel{4\int e^{-x} \sin 2x \, dx}$

$+ 4\int e^{-x} \sin 2x \, dx \qquad + 4\int e^{-x} \sin 2x \, dx$

$$\frac{5\int e^{-x} \sin 2x \, dx}{5} = \boxed{\frac{-e^{-x} \sin 2x - 2e^{-x} \cos 2x}{5} + C}$$

6.3

53) $\int \sin^{-1} x \, dx \quad = x\sin^{-1}x - {}^{-\frac{1}{2}}\int \overbrace{x \cdot \frac{1}{\sqrt{1-x^2}}\,dx \; [-2]}^{du}$

$u = \sin^{-1}x \qquad v = x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \quad dv = dx$

$u = 1 - x^2$

$du = -2x\,dx$

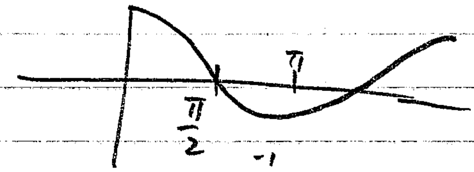
$x\sin^{-1}x + \frac{1}{2}\int u^{-1/2}\,du$

$x\sin^{-1}x + \cancel{\frac{1}{2}}\cdot\cancel{2}u^{1/2} + C$

$x\sin^{-1}x + \sqrt{1-x^2} + C$

25) $\int_0^{\pi/2} x^2 \sin 2x \, dx$

SIGN	u	dv
+	x^2	$\sin 2x$
−	$2x$	$-\frac{1}{2}\cos 2x$
+	2	$-\frac{1}{4}\sin 2x$
−	0	$\frac{1}{8}\cos 2x$



$\left. -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x \right|_0^{\pi/2}$

$\left[-\frac{1}{2}\left(\frac{\pi}{2}\right)^2 \overset{-1}{\cos 2\left(\frac{\pi}{2}\right)} + \cancel{\frac{1}{2}\left(\frac{\pi}{2}\right)\sin 2\left(\frac{\pi}{2}\right)} + \frac{1}{4}\overset{-1}{\cos 2\left(\frac{\pi}{2}\right)}\right] -$

$\left[\cancel{-\frac{1}{2}(0)^2\cos 2(0)} + \cancel{\frac{1}{2}(0)\sin 2(0)} + \frac{1}{4}\cos 2(0)\right]$

$\left[\frac{\pi^2}{8} - \frac{1}{4}\right] - \left[\frac{1}{4}\right] = \boxed{\frac{\pi^2}{8} - \frac{1}{2}}$

6.3

13) $\frac{du}{dx} = x\sec^2 x \quad , \quad u=1 \quad x=0$

$$\int du = \int x\sec^2 x \, dx$$

$$u = x \qquad v = \tan x$$
$$du = dx \qquad dv = \sec^2 x \, dx$$

$$u = x\tan x - \int \tan x \, dx$$

$$+ \int \frac{\sin x}{\cos x} dx \quad [-1]$$

$$u = \cos x$$
$$du = -\sin x \, dx$$

$$u = x\tan x + \int \frac{1}{u} du$$

$$u = x\tan x + \ln|u| + C$$

$$u = x\tan x + \ln|\cos x| + C$$

$$1 = 0 \cancel{\tan 0} + \ln|\cos 0| + C$$
$$\cancel{\ln 1}$$

$$1 = C$$

$$\boxed{u = x\tan x + \ln|\cos x| + 1}$$

6.3

17) $\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$

$u = \sin x \quad v = e^x \qquad\qquad u = \cos x \quad v = e^x$

$du = \cos x \, dx \quad dv = e^x dx \qquad\qquad du = -\sin x \, dx \quad dv = e^x dx$

$\int e^x \sin x \, dx = e^x \sin x - \left[e^x \cos x - \int -e^x \sin x \, dx \right]$

$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \cancel{\sin x} \, dx$

$+ \int e^x \sin x \, dx \qquad\qquad + \int e^x \cancel{\sin x} \, dx$

$\dfrac{2 \int e^x \sin x \, dx}{\cancel{2}} = \boxed{\dfrac{e^x \sin x - e^x \cos x}{2} + C}$

31) $\frac{dy}{d\theta} = \theta \sec^{-1}\theta \quad , \theta > 1 \qquad = \frac{1}{2}\theta^2 \sec^{-1}\theta - \frac{1}{2}\int \frac{1}{2}\theta^2 \cdot \frac{1}{\theta\sqrt{\theta^2-1}} \, d\theta$ du [2]

$\int dy = \int \theta \sec^{-1}\theta \, d\theta \qquad\qquad u = \theta^2 - 1$

$u = \sec^{-1}\theta \quad v = \frac{1}{2}\theta^2 \qquad\qquad du = 2\theta \, d\theta \qquad \frac{1}{\sqrt{u}}$

$du = \frac{1}{|\theta|\sqrt{\theta^2-1}} d\theta \quad dv = \theta \, d\theta \qquad\qquad u^{-1/2}$

$\frac{1}{2}\theta^2 \sec^{-1}\theta - \frac{1}{4}\int u^{-1/2} du$

$\frac{1}{2}\theta^2 \sec^{-1}(\theta) - \frac{1}{4} \cdot 2u^{1/2} + C$

$\frac{1}{2}\theta^2 \sec^{-1}(\theta) - \frac{1}{2}\sqrt{\theta^2 - 1} + C$

3) $\int 3te^{2t} dt = \frac{3}{2}te^{2t} - \int \frac{3}{2}e^{2t} dt$

$u = 3t \quad v = \frac{1}{2}e^{2t}$

$du = 3\,dt \quad dv = e^{2t} dt \qquad \boxed{\frac{3}{2}te^{2t} - \frac{3}{4}e^{2t} + C}$

11) $\frac{dy}{dx} = (x+2)\sin x \quad y = 2, x = 0 \qquad = -(x+2)\cos x - \int -\cos x \, dx$

$\int dy = \int (x+2)\sin x \, dx \qquad -(x+2)\cos x + \sin x + C = y$

$u = x+2 \quad v = -\cos x \qquad -(0+2)\cos 0 + \sin 0 + C = 2$

$du = dx \quad dv = \sin x \, dx \qquad -2 + C = 2$

$C = 4$

$\boxed{-(x+2)\cos x + \sin x + 4 = y}$

6.3

29) $\frac{dy}{dx} = x^2 e^{4x}$

$\int dy = \int x^2 e^{4x}\, dx$

$$\boxed{y = \frac{1}{4}x^2 e^{4x} - \frac{1}{8}x e^{4x} + \frac{1}{32}e^{4x} + C}$$

SIGN	u	dv
+	x^2	e^{4x}
−	$2x$	$\frac{1}{4}e^{4x}$
+	2	$\frac{1}{16}e^{4x}$
−	0	$\frac{1}{64}e^{4x}$

25) ANOTHER WAY

$\int_0^{\pi/2} x^2 \sin 2x\, dx \quad = \Big|_0^{\pi/2} -\frac{1}{2}x^2\cos 2x - \int -\frac{1}{\cancel{2}}\cos 2x\,(\cancel{2}x\,dx)$

$u = x^2 \qquad v = -\frac{1}{2}\cos 2x$

$du = 2x\,dx \quad dv = \sin 2x\, dx \qquad \Big|_0^{\pi/2} -\frac{1}{2}x^2\cos 2x + \int_0^{\pi/2} x\cos 2x\, dx$

$u = x \qquad v = -\frac{1}{2}\sin 2x$

$du = dx \quad dv = \cos 2x\, dx$

$$\Big|_0^{\pi/2} -\frac{1}{2}x^2\cos 2x + -\frac{1}{2}x\sin 2x - \int_0^{\pi/2} -\frac{1}{2}\sin 2x\, dx$$

$$\Big|_0^{\pi/2} -\frac{1}{2}x^2\cos 2x - \frac{1}{2}x\sin 2x - \frac{1}{4}\cos 2x$$

$$\int u\,dv = uv - \int v\,du$$

6.3

23) $\int x^3 e^{-2x}\,dx$

SIGN	u	dv
		e^{-2x}
$+$	x^3	$-\frac{1}{2}e^{-2x}$
$-$	$3x^2$	$\frac{1}{4}e^{-2x}$
$+$	$6x$	$-\frac{1}{8}e^{-2x}$
$-$	6	$\frac{1}{16}e^{-2x}$
$+$	0	

$$-\frac{1}{2}x^3e^{-2x} - \frac{3}{4}x^2e^{-2x} - \frac{3}{4}xe^{-2x} - \frac{3}{8}e^{-2x} + C$$

28) $\int_{-3}^{2} e^{-2x}\sin 2x\,dx = \Big|_{-3}^{2} -\frac{1}{2}e^{-2x}\sin 2x + \int_{-3}^{2} e^{-2x}\cos 2x\,dx$

$u = \sin 2x \quad v = -\frac{1}{2}e^{-2x}$

$du = 2\cos 2x\,dx \quad dv = e^{-2x}\,dx$

$u = \cos 2x \quad v = -\frac{1}{2}e^{-2x}$

$du = -2\sin 2x\,dx \quad dv = e^{-2x}\,dx$

$$\int_{-3}^{2} e^{-2x}\sin 2x\,dx = \Big|_{-3}^{2} -\frac{1}{2}e^{-2x}\sin 2x + -\frac{1}{2}e^{-2x}\cos 2x - \cancel{\int_{-3}^{2} e^{-2x}\sin 2x\,dx}$$

$$+\int_{-3}^{2} e^{-2x}\sin 2x\,dx \qquad\qquad +\cancel{\int_{-3}^{2} e^{-2x}\sin 2x\,dx}$$

$$\frac{\cancel{2}\int_{-3}^{2} e^{-2x}\sin 2x}{\cancel{2}} = \frac{\Big|_{-3}^{2} -\frac{1}{2}e^{-2x}\sin 2x - \frac{1}{2}e^{-2x}\cos 2x}{2}$$

$$\left[-\frac{1}{4}e^{-4}\sin 4 - \frac{1}{4}e^{-4}\cos 4\right] - \left[-\frac{1}{4}e^{6}\sin(-6) - \frac{1}{4}e^{6}\cos(-6)\right]$$