

6.4

$$31) \ln |y - y_c| = kt + C$$

$$\ln |60 - 20| = k(10) + \ln 70 \quad \downarrow$$

$$\ln |90 - 20| = k(0) + C$$

$$\ln 40 = 10k + \ln 70$$

$$-\ln 70 \quad -\ln 70$$

$$\underline{\ln 70 = C}$$

$$\frac{\ln \frac{4}{7}}{10} = \frac{10k}{10}$$

$$\ln |35 - 20| = \frac{\ln \frac{4}{7}}{10} t + \ln 70$$

$$-\ln 70 \quad -\ln 70$$

$$\ln \frac{15}{70} = \frac{\ln \frac{4}{7}}{10} t$$

$$\frac{10 \ln \frac{15}{70}}{\ln \frac{4}{7}} = t \approx 27.527 - 10$$

17.527 minutes
longer

$$(b) \quad k = \frac{\ln \frac{4}{7}}{10}$$

$$\ln |90 - 15| = k(0) + C$$

$$\ln 105 = C$$

$$\ln |35 - 15| = \frac{\ln \frac{4}{7}}{10} t + \ln 105$$

$$\ln 50 = \frac{\ln \frac{4}{7}}{10} t + \ln 105$$

$$-\ln 105 \quad -\ln 105$$

$$\ln \frac{10}{21} = \frac{\ln \frac{4}{7}}{10} t$$

$$\frac{10 \ln \frac{10}{21}}{\ln \frac{4}{7}} = t \approx 13.258$$

minutes

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2) $\frac{dy}{dt} = -0.0077y$

$$\frac{dy}{dt} = ky$$

$$\int \frac{1}{y} dy = \int -0.0077 dt$$

$$\ln|y| = -0.0077t + C$$

$$e^{-0.0077t+C} = y$$

$$e^{-0.0077t} e^C = y$$

$$\frac{e^{-0.0077t}}{e^{-0.0077t}} = \frac{y}{y} \leftarrow$$

$$e^{-0.0077t} = \frac{1}{2}$$

$$\ln e^{-0.0077t} = \ln \frac{1}{2}$$

$$\frac{-0.0077t}{-0.0077} = \frac{\ln \frac{1}{2}}{-0.0077}$$

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7) $\frac{dy}{dx} = (\cos x) e^{y+\sin x}$

$$\frac{dy}{dx} = (\cos x) e^y e^{\sin x}$$

$$\int \frac{1}{e^y} dy = \int (\cos x) e^{\sin x} dx \quad du$$

$$u = -y \quad \frac{du}{dy} = -1 \quad \int e^{-y} dy \quad u = \sin x \quad \frac{du}{dx} = \cos x$$

$$-e^{-y} = \int e^u du$$

$$-e^{-y} = e^u + C$$

$$-e^{-y} = e^{\sin x} + C$$

$$e^{-y} = -e^{\sin x} + C$$

$$-y = \ln |-e^{\sin x} + C|$$

$$y = -\ln |-e^{\sin x} + C|$$

$$22) \frac{dy}{dt} = -ky \rightarrow y = \frac{C e^{-kt}}{C}$$

$$\frac{1}{2} = e^{-kt}$$

$$\frac{1}{2} = e^{-k(65)}$$

$$\ln \frac{1}{2} = \ln e^{-65k}$$

$$\frac{\ln \frac{1}{2}}{65} = \frac{-65k}{65}$$

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$$5) \frac{dy}{dx} = (y+5)(x+2) \quad y=1 \text{ when } x=0$$

$$\int \frac{1}{y+5} dy = \int (x+2) dx$$

$$\ln|y+5| = \frac{1}{2}x^2 + 2x + C \rightarrow \ln|y+5| = \frac{1}{2}x^2 + 2x + \ln 6$$

$$\ln 6 = \frac{1}{2}(0)^2 + 2(0) + C$$

$$\ln 6 = C$$

$$y+5 = e^{\frac{1}{2}x^2 + 2x + \ln 6}$$

$$y = e^{\frac{1}{2}x^2 + 2x} \cdot e^{\ln 6} - 5$$

$$y = e^{\frac{1}{2}x^2 + 2x} \cdot 6 - 5$$

$$y = 6e^{\frac{1}{2}x^2 + 2x} - 5$$

$$7) \frac{dy}{dx} = (\cos x) e^{y+\sin x} \quad y=0, x=0$$

$$\frac{dy}{dx} = (\cos x) e^y e^{\sin x}$$

$$\int e^{-y} dy = \int (\cos x) e^{\sin x} dx \quad du$$

$$u = \sin x$$

$$-e^{-y} = du = \cos x dx$$

$$-e^{-y} = \int e^u du$$

$$-e^{-y} = e^u + C$$

$$-e^{-y} = e^{\sin x} + C \rightarrow -e^{-y} = e^{\sin x} - 2$$

$$-e^{-0} = e^{\sin(0)} + C$$

$$e^{-y} = 2 - e^{\sin x}$$

$$-1 = 1 + C$$

$$-y = \ln(2 - e^{\sin x})$$

$$-2 = C$$

$$y = -\ln(2 - e^{\sin x})$$

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11) $\frac{dy}{dt} = ky$

$$y = C e^{kt}$$

$$y = 1000 e^{.15t}$$

13) $y = 50 e^{skt} \rightarrow y = 50 e^{\frac{\ln 2}{5} \cdot t}$
 $100 = 50 e^{5k}$
 $2 = e^{5k}$
 $\frac{\ln 2}{5} = \frac{5k}{5}$

17) $A = P e^{rt}$
 $A = P e^{.0525t}$
 $2898.44 = P e^{.0525(30)}$

$$\frac{2898.44}{e^{.0525(30)}} = \frac{P}{e^{-.0525(30)}}$$

$$\boxed{\$600 = P}$$

$$\frac{A}{P} = e^{.0525t} = 2$$

$$\ln e^{.0525t} = \ln 2$$

$$.0525t = \frac{\ln 2}{.0525}$$

$$\boxed{t \approx 13.203 \text{ years}}$$

35) HALF-LIFE OF CARBON-14 IS 5700 YEARS

$$\frac{y}{c} = e^{kt} = \frac{1}{2}$$

$$e^{5700k} = \frac{1}{2}$$

$$\ln e^{5700k} = \ln \frac{1}{2}$$

$$\frac{5700k}{5700} = \frac{\ln \frac{1}{2}}{5700}$$

$$\frac{y}{c} = e^{\frac{\ln \frac{1}{2}}{5700} \cdot t} = .445$$

$$\frac{5700}{\ln \frac{1}{2}} \cdot \frac{\ln \frac{1}{2}}{5700} t = \ln .445 \cdot \frac{5700}{\ln \frac{1}{2}}$$

$$t = 6658 \text{ years}$$

3) $\frac{dy}{dx} = \frac{y}{x}$ $y=2$ when $x=2$

$$\int \frac{1}{y} dy = \int \frac{1}{x} dx$$

$$\ln |y| = \ln |x| + C \rightarrow \ln |y| = \ln |x|$$

$$\ln 2 = \ln 2 + C$$

$$0 = C$$

$$\boxed{y = x}$$

25) $y = y_0 e^{-.018t}$

$$\frac{y}{y_0} = e^{-.018t} = .9$$

$$-.018t = \ln .9$$

$$\frac{-.018t}{-.018} = \frac{\ln .9}{-.018}$$

$$t = .585 \text{ days}$$

21) $\frac{dy}{dt} = -0.00777y$ $\frac{dy}{dt} = ky$ $y = Ce^{kt}$

$$\frac{y}{C} = e^{-0.00777t} = \frac{1}{2}$$

$$-0.00777t = \ln \frac{1}{2}$$

$$\frac{-0.00777t}{-0.00777} = \frac{\ln \frac{1}{2}}{-0.00777}$$

$$t = 90.019 \text{ years}$$

$$k = \frac{\ln \frac{1}{2}}{\text{half-life}}$$

$$\text{half-life} = \frac{\ln \frac{1}{2}}{k}$$

29) $y = y_0 e^{-kt}$ $t = \frac{3}{k}$

$$y = y_0 e^{-k(\frac{3}{k})}$$

$$y = y_0 e^{-3}$$

$$y = y_0 e^{-3} = .049$$

4.9% left

y_0 y

23) $y = Ce^{kt}$

$$\frac{y}{C} = e^{kt}$$

$$\frac{y}{C} = e^{kt} = 2$$

$$e^{k(1.5)} = 2$$

$$.5k = \ln 2$$

$$k = 2 \ln 2$$

$$y = e^{2 \ln 2 t}$$

$$y = e^{2 \ln 2 (29)}$$

$$y = 2.815 \times 10^{14}$$

281,500,000,000,000

1) $\frac{dy}{dx} = \frac{x}{y}$ $y=2, x=1$

$$\int y dy = \int x dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C \rightarrow \frac{1}{2}y^2 = \frac{1}{2}x^2 + \frac{3}{2}$$

$$\frac{1}{2}(2)^2 = \frac{1}{2}(1)^2 + C$$

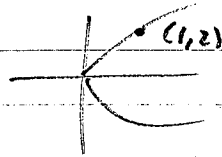
$$2 = \frac{1}{2} + C$$

$$\frac{3}{2} = C$$

$$y^2 = x^2 + 3$$

$$y = \pm \sqrt{x^2 + 3}$$

$$y = \sqrt{x^2 + 3}$$



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$$(a) A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$(b) \frac{A}{P} = \left(1 + \frac{r}{n}\right)^{nt}$$

$$(a) A = 2000 \left(1 + \frac{.0475}{1}\right)^{1 \cdot t}$$

$$2 = \left(1 + \frac{.0475}{12}\right)^{12t}$$

$$4000 = 2000 \left(1 + \frac{.0475}{1}\right)^t$$

$$2 = (1.0475)^t$$

$$\ln 2 = \frac{t \ln(1.0475)}{\ln(1.0475)}$$

$$\ln(1.0475)$$

$$= t$$

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$$30) \ln |y - y_c| = kt + C$$

$$\ln |35 - 65| = k(10) + C \quad \ln |50 - 65| = k(20) + C$$

$$\ln 30 = 10k + C$$

$$\ln 15 = 20k + C$$

$$-20k \quad -20k$$

$$\ln 30 = 10k + \ln 15 - 20k$$

$$\ln 15 - 20k = C$$

$$-\ln 15 \quad -\ln 15$$

$$\ln 15 - 20 \left(\frac{\ln 2}{-10} \right) = C$$

$$\boxed{\frac{\ln 2}{-10}} = \frac{-10k}{-10}$$

$$\ln 15 + 2 \ln 2 = C$$

$$\ln 15 + \ln 2^2 = C$$

$$\ln 60 = C$$

$$\ln |y - 65| = \frac{\ln 2}{-10} t + \ln 60$$

$$\ln |y - 65| = \ln 60$$

$$y - 65 = 60$$

$$y - 65 = -60$$

$$y = 125$$

$$\boxed{y = 5^\circ \text{F}}$$

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$$3) \ln |y - y_c| = kt + C$$

$$\ln |60 - 20| = k(10) + \ln 70$$

$$\ln |90 - 20| = k(0) + C$$

$$\begin{aligned} \ln 40 &= 10k + \ln 70 \\ -\ln 70 &\quad -\ln 70 \end{aligned}$$

$$\ln 70 = C$$

$$\frac{\ln \frac{4}{7}}{10} = \frac{10k}{10}$$

$$\ln |y - 20| = \frac{\ln \frac{4}{7}}{10} t + \ln 70$$

$$(a) \ln |35 - 20| = \frac{\ln \frac{4}{7}}{10} t + \ln 70$$

$$\begin{aligned} \ln 15 &= \frac{\ln \frac{4}{7}}{10} t + \ln 70 \\ -\ln 70 &\quad -\ln 70 \end{aligned}$$

$$\ln \frac{3}{14} = \frac{\ln \frac{4}{7}}{10} t$$

$$\frac{10 \ln \frac{3}{14}}{\ln \frac{4}{7}} = 10$$

$$\frac{10 \ln \frac{3}{14}}{\ln \frac{4}{7}} = t$$

$$(b) \ln |90 - -15| = \frac{\ln \frac{4}{7}}{10} (0) + C$$

$$\ln |105| = C$$

$$\ln |35 - -15| = \frac{\ln \frac{4}{7}}{10} t + \ln 105$$

$$\begin{aligned} \ln 50 &= \frac{\ln \frac{4}{7}}{10} t + \ln 105 \\ -\ln 105 &\quad -\ln 105 \end{aligned}$$

$$\ln \frac{10}{21} = \frac{\ln \frac{4}{7}}{10} t$$

$$\frac{10 \ln \frac{10}{21}}{\ln \frac{4}{7}} = t$$

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$$31) \ln |y - y_c| = kt + C$$

$$\ln |60 - 20| = k(10) + \ln 70$$

$$\ln |90 - 20| = k(0) + C$$

$$\begin{array}{r} \ln 40 = 10k + \ln 70 \\ -\ln 70 \qquad \qquad -\ln 70 \end{array}$$

$$\ln 70 = C$$

$$\rightarrow \frac{\ln \frac{4}{7}}{10} = \frac{10k}{10}$$

$$(a) \ln |y - 20| = \frac{\ln \frac{4}{7}}{10} t + \ln 70$$

$$\ln |35 - 20| = \frac{\ln \frac{4}{7}}{10} t + \ln 70$$

$$\begin{array}{r} \ln 15 = \frac{\ln \frac{4}{7}}{10} t + \ln 70 \\ -\ln 70 \qquad \qquad -\ln 70 \end{array}$$

$$\frac{10 \ln 15}{\ln \frac{4}{7}} - 10$$

$$\ln \frac{3}{14} = \frac{\ln \frac{4}{7}}{10} t$$

$$\frac{10 \ln \frac{3}{14}}{\ln \frac{4}{7}} = t$$

$$(b) \ln |90 - -15| = k(0) + C$$

$$\ln |y + 15| = \frac{\ln \frac{4}{7}}{10} t + \ln 105$$

$$\ln 105 = C$$

$$\ln |35 + 15| = \frac{\ln \frac{4}{7}}{10} t + \ln 105$$

$$\begin{array}{r} \ln 50 = \frac{\ln \frac{4}{7}}{10} t + \ln 105 \\ -\ln 105 \qquad \qquad -\ln 105 \end{array}$$

$$\ln \frac{10}{21} = \frac{\ln \frac{4}{7}}{10} t$$

$$\frac{10 \ln \frac{10}{21}}{\ln \frac{4}{7}} = t$$