8.4

35) $\int_0^{\ln 2} y^{-2} e^{1/y}\, dy$ $\qquad u = \frac{1}{y} = y^{-1}$

$du = -y^{-2}$

~~$\lim_{a \to 0^+} \int_a^{\ln 2} y^{-2} e^{1/y}\, dy$~~

$\lim_{a \to 0^+} -\int_a^{\ln 2} \underbrace{-y^{-2}}_{du} e^{1/y}\, dy$ $\qquad \int e^u\, du$

$\qquad | e^u$

$\lim_{a \to 0^+} - [redacted] \Big|_a^{\ln 2} e^{1/y}$

$\lim_{a \to 0^+} -e^{1/\ln 2} + e^{1/a}$ DIVERGENT

$\frac{1}{1 + (\sqrt{s})^2} \cdot \frac{1}{\sqrt{s}}$

37) $\int_0^{\infty} \frac{ds}{(1+s)\sqrt{s}} \to \int_0^1 \frac{ds}{(1+s)\sqrt{s}} + \int_1^{\infty} \frac{ds}{(1+s)\sqrt{s}}$

$\lim_{a \to 0^+} 2\int_a^1 \frac{\frac{1}{2}\, ds}{(1+s)\sqrt{s}} + \lim_{b \to \infty} 2 [redacted] \int_1^b \frac{ds}{(1+s)\sqrt{s}}$ $\qquad u = s^{1/2}$

$du = \frac{1}{2} s^{-1/2} = \frac{1}{2\sqrt{s}}$

$\lim_{a \to 0^+} \Big|_a^1 2\arctan\sqrt{s} + \lim_{b \to \infty} \Big|_1^b 2\arctan\sqrt{s}$

$\lim_{a \to 0^+} [2\arctan\sqrt{1} - 2\arctan\sqrt{a}] + \lim_{b \to \infty} [2\arctan\sqrt{b} - 2\arctan\sqrt{1}]$

$[\cancel{2 \cdot \frac{\pi}{4}} - 0] + [2 \cdot \frac{\pi}{2} - \cancel{2 \cdot \frac{\pi}{4}}] = \pi$

11) $\int_{-\infty}^{-2} \frac{2\, dx}{x^2 - 1}$ $\qquad (x-1)^2 \qquad \frac{2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

$\lim_{a \to -\infty} \int_a^{-2} \frac{2\, dx}{x^2 - 1}$ $\qquad A(x+1) + B(x-1) = 2$

$x = 1 \qquad 2A = 2 \to A = 1$

$x = -1 \qquad -2B = 2 \to B = -1$

$\lim_{a \to -\infty} \int_a^{-2} \left(\frac{1}{x-1} - \frac{1}{x+1} \right) dx$

$\lim_{a \to -\infty} \Big|_a^{-2} \ln|x-1| - \ln|x+1|$

$\lim_{a \to -\infty} \left[\ln \frac{-2-1}{-2+1} \right] - \left[\cancel{\ln \frac{a-1}{a+1}} \right] = \ln 3$

8.4

23) $\int_{-\infty}^{\infty} \frac{dx}{e^x+e^{-x}} \to \int_{-\infty}^{0} \frac{dx}{e^x+e^{-x}} + \int_{0}^{\infty} \frac{dx}{e^x+e^{-x}}$

$$\lim_{a\to-\infty} \int_a^0 \frac{dx}{e^x+e^{-x}} + \lim_{b\to\infty} \int_0^b \frac{dx}{e^x+e^{-x}}$$

$$\lim_{a\to-\infty} \Big|_a^0 \arctan e^x + \lim_{b\to\infty} \Big|_0^b \arctan e^x$$

$$\lim_{a\to-\infty} \left[\tan^{-1} e^0 - \tan^{-1} e^a\right] + \lim_{b\to\infty} \left[\tan^{-1} e^b - \tan^{-1} e^0\right]$$

$$\left[\cancel{\frac{\pi}{4}} - 0\right] + \left[\frac{\pi}{2} - \cancel{\frac{\pi}{4}}\right] = \frac{\pi}{2}$$

$$\frac{1}{e^{-x}(e^{2x}+1)}$$

$$\frac{e^x}{e^{2x}+1}$$

$$\frac{e^x}{(e^x)^2+1} \qquad \frac{1}{u^2+1}\,du$$

25) $\int_0^2 \frac{dx}{1-x^2} \to \int_0^1 \frac{dx}{1-x^2} + \int_1^2 \frac{dx}{1-x^2}$ $\qquad \frac{1}{1-x^2} = \frac{1}{(1-x)(1+x)}$

$A(1-x)+B(1+x)=1$

$$\lim_{b\to1^-} \int_0^b \frac{dx}{1-x^2} + \lim_{a\to1^+} \int_a^2 \frac{dx}{1-x^2}$$

$x=-1 \qquad 2A=1 \to A=\frac{1}{2}$

$$\lim_{b\to1^-} \int_0^b \left(\frac{1}{2(1-x)} + \frac{1}{2(1+x)}\right) dx + \lim_{a\to1^+} \int_a^2$$

DITTO

$x=1 \qquad 2B=1 \to B=\frac{1}{2}$

$$\lim_{b\to1^-} \Big|_0^b \left(-\frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x|\right) + \lim_{a\to1^+} \Big|_a^2 \left(-\frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x|\right)$$

$$\lim_{b\to1^-} \Big|_0^b \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + \lim_{a\to1^+} \Big|_a^2 \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$$