

9.1
 17) $\sum_{n=0}^{\infty} \sin^n \left(\frac{\pi}{4} + n\pi \right)$

$$\left(\frac{1}{\sqrt{2}} \right)^0 + \left(-\frac{1}{\sqrt{2}} \right)^1 + \left(\frac{1}{\sqrt{2}} \right)^2$$

$$\sum_{n=0}^{\infty} \left(-\frac{1}{\sqrt{2}} \right)^n \quad \frac{1}{1 - \frac{1}{\sqrt{2}}}$$

$$\left(\frac{1}{\sqrt{2}} \right)^0 + \left(-\frac{1}{\sqrt{2}} \right)^1 + \left(\frac{1}{\sqrt{2}} \right)^2 + \dots + \left(-\frac{1}{\sqrt{2}} \right)^n + \dots$$

21) $\sum_{n=0}^{\infty} 2^n x^n = \sum_{n=0}^{\infty} (2x)^n \quad |2x| < 1 \quad \left(-\frac{1}{2}, \frac{1}{2} \right)$

$$|x| < \frac{1}{2}$$

$$|x - c| < R$$

$$c = 0 \quad R = \frac{1}{2}$$

$$\sum_{n=0}^{\infty} 2^n x^n = \frac{1}{1 - 2x}$$

27) $\sum_{n=0}^{\infty} 2^n n x^{n-1} = \frac{2}{(1-2x)^2}$

$$(1-2x)^{-1}$$

$$- (1-2x)^{-2} [-2]$$

31) $\sum_{n=0}^{\infty} \frac{2^n}{n+1} x^{n+1} = -\frac{1}{2} \ln |1-2x|$

$$\int \frac{1}{1-2x}$$

$$-\frac{1}{2} \ln |1-2x|$$

$$23) \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n (x-3)^n = \sum_{n=0}^{\infty} \left(-\frac{1}{2} \cdot (x-3)\right)^n$$

$$\frac{1}{1 - \left(-\frac{1}{2}(x-3)\right)} \quad \left| +\frac{1}{2}(x-3) \right| < 1$$

$$\frac{1}{1 + \frac{1}{2}x - \frac{3}{2}}$$

$$|x-3| < 2$$

$$\boxed{(1, 5)}$$

$$\frac{1}{\frac{1}{2}x - \frac{1}{2}} = \boxed{\frac{2}{x-1}}$$

$$\frac{1}{\frac{1}{2}(x-1)}$$

$$\frac{d}{dx} = 2(x-1)^{-1} - 2(x-1)^{-2}$$

$$29) \sum_{n=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^n}{n} (x-3)^{n-1} = \frac{-2}{(x-1)^2}$$

$$\rightarrow x + x^2 + x^3 + x^4 + \dots + x^{n+1} + \dots$$

$$1 + 2x + 3x^2 + 4x^3 + \dots + (n+1)x^n + \dots$$

$$\downarrow 2 + 6x + 12x^2 + \dots + n(n+1)x^{n-1}$$

$$33) \sum_{n=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^n}{n+1} (x-3)^{n+1} = 2 \ln|x-1| \quad \int \frac{2(x-1)}{x-1} dx$$

$$53) \sum_{n=1}^{\infty} ar^{n-1} \xrightarrow{\downarrow} \frac{a(1-r^n)}{1-r} = S_n = \frac{a-ar^n}{1-r}$$

$$r < 1 \quad \lim_{n \rightarrow \infty} \frac{a(1-r^n)}{1-r} = \frac{a}{1-r}$$

$$r > 1 \quad \lim_{n \rightarrow \infty} \frac{a(1-r^n)}{1-r} = \infty$$

$$71) \sum_{n=0}^{\infty} (x-1)^n = \frac{1}{1-(x-1)} = \frac{1}{2-x}$$

$$\int \frac{1}{2-x}$$

$$-\ln |2-x| + C$$

$$-\ln C |2-x|$$

$$-\ln \frac{|2-x|}{C}$$