

$$f(x) = e^x \quad g(x) = x^2$$

$$f(g(x)) = e^{x^2}$$

9.2

$$25) \quad e^x = 1 + x + \frac{1}{2!} x^2 + \dots + \frac{1}{n!} x^n + \dots = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

$$(a) \quad e^{x/2} = 1 + \frac{x}{2} + \frac{1}{2!} \left(\frac{x}{2}\right)^2 + \dots + \frac{1}{n!} \left(\frac{x}{2}\right)^n + \dots = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{x}{2}\right)^n$$

~~$$(b) \quad e^{x/2} - 1 = 1 + \frac{x}{2} + \frac{1}{2!} \left(\frac{x}{2}\right)^2 + \frac{1}{3!} \left(\frac{x}{2}\right)^3 + \dots + \frac{1}{n!} \left(\frac{x}{2}\right)^n - 1$$~~

~~$$\frac{e^{x/2} - 1}{x} = \frac{\frac{x}{2}}{x} + \frac{\frac{1}{2!} \left(\frac{x}{2}\right)^2}{x} + \dots + \frac{\frac{1}{n!} \left(\frac{x}{2}\right)^n}{x}$$~~

~~$$= \frac{1}{2} + \frac{1}{2!} \frac{x}{4} + \frac{1}{3!} \frac{x^2}{8} + \dots +$$~~

~~$$e^x - 1 = 1 + x + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 + \dots$$~~

~~$$\frac{e^x - 1}{x} = \frac{x}{x} + \frac{1}{2!} \frac{x^2}{x} + \frac{1}{3!} \frac{x^3}{x} + \dots$$~~

~~$$= 1 + \frac{1}{2!} x + \frac{1}{3!} x^2 + \dots = \sum_{n=1}^{\infty} \frac{1}{n!} x^{n-1}$$~~

$$(c) \quad \frac{d}{dx} \frac{e^x - 1}{x} = \frac{e^x \cdot x - (e^x - 1)}{x^2}$$

$$= \frac{x e^x - e^x + 1}{x^2}$$

$$\frac{d}{dx} \sum_{n=1}^{\infty} \frac{x^{n-1}}{n!}$$

$$\sum_{n=1}^{\infty} \frac{(n-1)x^{n-2}}{n!}$$

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)!} x^n$$

$$\sum_{n=0}^{\infty} \frac{n}{(n+1)!} x^{n-1} = \frac{e^1 - e^1 + 1}{1^2} = 1$$

$$\sum_{n=1}^{\infty} \frac{n-1}{n!} = 1$$

9.2

1)  $f(x) = \sqrt{1+x^2}$

$g_0(x) = (1+x)^{1/2}$	$g_0(0) = 1$
$g_1(x) = \frac{1}{2}(1+x)^{-1/2}$	$g_1(0) = \frac{1}{2}$
$g_2(x) = \frac{-1}{4}(1+x)^{-3/2}$	$g_2(0) = -\frac{1}{4}$
$g_3(x) = \frac{3}{8}(1+x)^{-5/2}$	$g_3(0) = \frac{3}{8}$
$g_4(x) = \frac{-15}{16}(1+x)^{-7/2}$	$g_4(0) = -\frac{15}{16}$

$$1 + \frac{1}{2}x + \frac{-1}{4 \cdot 2!}x^2 + \frac{3}{8 \cdot 3!}x^3 - \frac{15}{16 \cdot 4!}x^4$$

$1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{1}{16}x^6$

11.  $\frac{1}{1-x} (1-x)^{-1}$