9.4

35) $\lim_{n\to\infty} \left| \frac{(n+1)^2 e^{-(n+1)}}{n^2 e^{-n}} \right|$ ← $-n-1$

$\lim_{n\to\infty} n^2 e^{-n} = \lim_{n\to\infty} \frac{n^2}{e^n} = 0$

$\lim_{n\to\infty} \left| \frac{(n+1)^2 e^{-1}}{n^2} \right|$

$e^{-1} = \frac{1}{e} < 1$ CONVERGES

49) $\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)} = \sum_{n=1}^{\infty} \frac{A}{2n-1} + \frac{B}{2n+1} = \sum_{n=1}^{\infty} \frac{3}{2n-1} - \frac{3}{2n+1}$

$A(2n+1) + B(2n-1) = 6$

$\left(\frac{3}{1} - \frac{3}{3}\right) + \left(\frac{3}{3} - \frac{3}{5}\right) + \left(\frac{3}{5} - \frac{3}{7}\right)$

$n = \frac{1}{2}$ $2A = 6 \to A = 3$

$\lim_{n\to\infty} 3 - \frac{3}{2n+1} = 3$

$n = -\frac{1}{2}$ $-2B = 6 \to B = -3$

51) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} = \sum_{n=1}^{\infty} \frac{A}{n} + \frac{B}{n^2} + \frac{C}{n+1} + \frac{D}{(n+1)^2}$

$A(n)(n+1)^2 + B(n+1)^2 + Cn^2(n+1) + Dn^2 = 2n+1$

$n = 0$ $B = 1$ $\sum_{n=1}^{\infty} \frac{1}{n^2} - \frac{1}{(n+1)^2}$ $D(-1)^2 = 2(-1)+1$

$D = -1$

$n = -1$ $D = -1$

$\left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{9}\right) + \cdots$

$\lim_{n\to\infty} 1 - \frac{1}{(n+1)^2} = 1$

$\sum 4(1)^n$

9.4

25) $\sum_{n=0}^{\infty} \left(\frac{\sqrt{x}}{2} - 1\right)^n$

$\frac{\sqrt{x}}{2} - 1 < 1$

$\frac{\sqrt{x}}{2} < 2$

$\sqrt{x} < 4$

$|x| < 16$

$(-16, 16)$

$(0, 16)$

$\frac{1}{1 - \left(\frac{\sqrt{x}}{2} - 1\right)}$

27) $\sum_{n=0}^{\infty} \left(\frac{x^2 - 1}{3}\right)^n$

$\frac{x^2 - 1}{3} < 1$

$x^2 - 1 < 3$

$x^2 < 4$

$|x| < 2$

$(-2, 2)$

$\frac{a_1}{1 - r}$

$\frac{1}{1 - \left(\frac{x^2 - 1}{3}\right)}$

31) $\sum_{n=1}^{\infty} \frac{n^2 - 1}{2^n}$

$\lim_{n \to \infty} \left| \frac{(n+1)^2 - 1}{2^{n+1}} \cdot \frac{2^n}{n^2 - 1} \right|$

$\lim_{n \to \infty} \left| \frac{(n+1)^2 - 1}{n^2 - 1} \cdot \frac{1}{2} \right|$

$\frac{n^2 + 2n + 1}{2n^2 - 2}$

$\frac{1}{2}$ CONVERGES

33) $\sum_{n=1}^{\infty} \frac{2^n}{3^{n+1}}$

$\lim_{n \to \infty} \left| \frac{2^{n+1}}{3^{n+1} + 1} \cdot \frac{3^n + 1}{2^n} \right|$

$\frac{2}{3}$ CONVERGES

9.4

29) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ $\lim_{n\to\infty} \frac{n}{n+1} = 1$ DIVERGES

19) $\sum_{n=0}^{\infty} (-2)^n (n+1)(x-1)^n$ $\lim_{n\to\infty} \left| \frac{(-2)^{n+1}(n+2)(x-1)^{n+1}}{(-2)^n (n+1)(x-1)^n} \right| < 1$

$\lim_{n\to\infty} \left| 2 \frac{(n+2)}{(n+1)} (x-1) \right| < 1$

$|2(x-1)| < 1$

$R = \frac{1}{2}$ $|x-1| < \frac{1}{2}$

13) $\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}\,3^n}$ $\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)\sqrt{n+1}\,3^{n+1}} \cdot \frac{n\sqrt{n}\,3^n}{x^n} \right| < 1$

$\lim_{n\to\infty} \left| \frac{n\sqrt{n}}{3(n+1)\sqrt{n+1}} x \right|$

$\left| \frac{x}{3} \right| < 1$

$R = 3$

$|x| < 3$

43) $\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$ $\lim_{n\to\infty} \left| \frac{(n+1)!}{(2n+3)!} \cdot \frac{(2n+1)!}{n!} \right|$

$\lim_{n\to\infty} \left| \frac{n+1}{(2n+3)(2n+2)} \right|$

0 CONVERGES