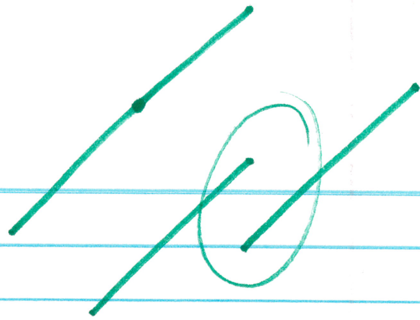
3.1

$x=1$

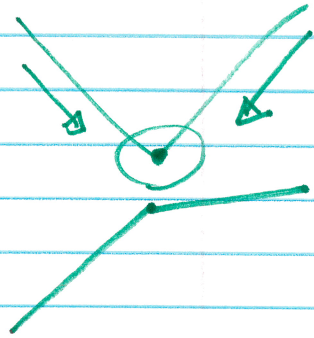
(3) $\lim_{h\to0^-} \frac{[(1+h)^2+(1+h)]-[1^2+1]}{h}$

$\lim_{h\to0^-} \frac{[\cancel{1}+2h+h^2+\cancel{1}+h]-[\cancel{2}]}{h} = \lim_{h\to0^-}$

$\lim_{h\to0^-} \frac{\cancel{h}(3+h)}{\cancel{h}} = 3$



$\lim_{h\to0^+} \frac{[3(1+h)-2]-[1^2+1]}{h}$



$\lim_{h\to0^+} \frac{[3+3h-2]-[2]}{h}$

$\lim_{h\to0^+} \frac{-1+3h}{h} = \lim_{h\to0^+} -\frac{1}{h}+\frac{3h}{h}$

$-\infty \qquad 3$

$f(x)=\frac{1}{x}$

$f(2)=\frac{1}{2}$

$f(a=2)=$

(5) $\lim_{x\to2} \frac{2\cdot\frac{1}{x}-\frac{1}{2}\cdot x}{x-2}$

$\lim_{x\to2} \frac{\frac{2}{2x}-\frac{x}{2x}}{x-2} = \lim_{x\to2} \frac{\frac{2-x}{2x}}{x-2 \quad -(x-2)}$

$\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ $a\to2$, $x\to2$

$\lim_{x\to2} \frac{2-x}{2x}\cdot\frac{1}{\cancel{x-2}}$

$\lim_{x\to2} -\frac{1}{2x} = \boxed{-\frac{1}{4}}$

3.1

(7) $$\lim_{x \to 3} \frac{\sqrt{x+1} - \sqrt{3+1}}{x-3}$$

$$\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2}$$

$$\lim_{x \to 3} \frac{(x+1) \cancel{+ 2\sqrt{x+1}} \cancel{- 2\sqrt{x+1}} - 4}{(x-3)\left[\sqrt{x+1} + 2\right]}$$

$$\lim_{x \to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}\left[\sqrt{x+1} + 2\right]} \qquad \frac{1}{4}$$

3.1

(1) $f(x)=\frac{1}{x}$, $a=2$

$$\lim_{h\to 0}\frac{\left[\frac{1}{2+h}\right]-\left[\frac{1}{2}\right]}{h}$$

(20) $y=\sqrt{x}$ @ $x=4$ $(4,2)$

$$\lim_{h\to 0}\frac{\left[\sqrt{4+h}\right]-\left[\sqrt{4}\right]}{h}$$

$$\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}\cdot\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$$

$$\lim_{h\to 0}\frac{\cancel{4}+h-\cancel{4}}{h(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0}\frac{1}{\sqrt{4+h}+2}=\frac{1}{4}$$

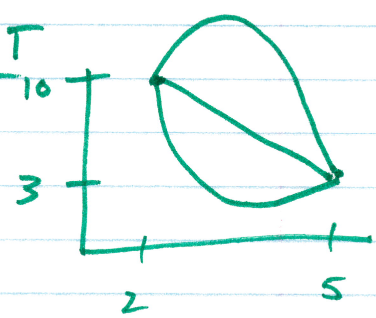
TANGENT

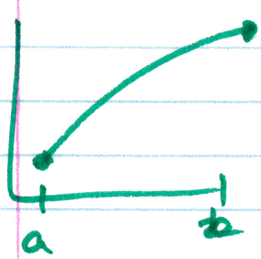
$$y-2=\frac{1}{4}(x-4)$$

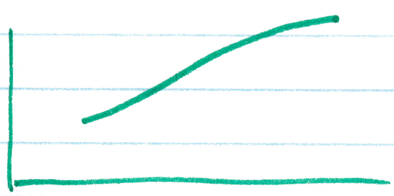
NORMAL

$$y-2=-4(x-4)$$

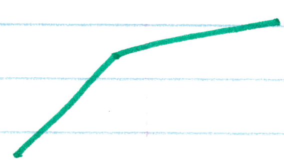
IVT







5



$f'(2)=1$

$f'(6)=6$

3.1

7 $\sqrt{x+1}$, $a=3$

$$\lim_{x\to 3} \frac{\left[\sqrt{x+1}\right]-\left[\sqrt{3+1}\right]}{x-3}$$

$$\lim_{x\to 3} \frac{\left(\sqrt{x+1}-2\right)}{x-3}\cdot\frac{\left(\sqrt{x+1}+2\right)}{\sqrt{x+1}+2}$$

$$\lim_{x\to 3} \frac{(x+1)+\cancel{2\sqrt{x+1}}-\cancel{2\sqrt{x+1}}-4}{(x-3)\left(\sqrt{x+1}+2\right)}$$

$$\lim_{x\to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}\left(\sqrt{x+1}+2\right)} = \lim_{x\to 3} \frac{1}{\sqrt{x+1}+2} = \boxed{\frac{1}{4}}$$

17 $f(2)=3$, $f'(2)=5$

(a) $(2,3)$ $m=5$ $\boxed{y-3=5(x-2)}$

(b) $(2,3)$ $m=-\frac{1}{5}$ $\boxed{y-3=-\frac{1}{5}(x-2)}$

5 $\frac{1}{x}$, $a=2$

$$\lim_{x\to 2} \frac{\left[\frac{1}{x}-\frac{1}{2}\right]2x}{(x-2)2x} = \lim_{x\to 2} \frac{(2-x)}{(x-2)2x} = \lim_{x\to 2} \frac{-\cancel{(-2+x)}}{\cancel{(x-2)}2x}$$

$$\lim_{x\to 2} -\frac{1}{2x} = \boxed{-\frac{1}{4}}$$

3.1

(19) $y = x^3$ $\quad (1, 1)$

$$\lim_{x \to 1} \frac{x^3 - 1^3}{x - 1} = \lim_{x \to 1} \frac{x^3 - 1}{x - 1}$$

$$\begin{array}{r} x^2 + x + 1 \\ x - 1 \overline{) x^3 + 0x^2 + 0x - 1} \\ -(x^3 - x^2) \\ \hline x^2 + 0x \\ -(x^2 - x) \\ \hline x - 1 \\ -(x - 1) \\ \hline 0 \end{array}$$

$$\lim_{x \to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}$$

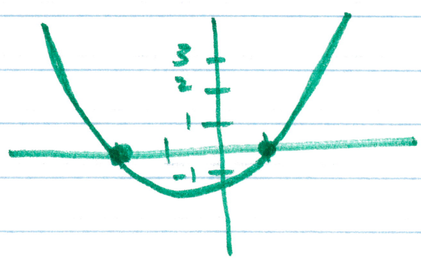
$$\lim_{x \to 1} x^2 + x + 1 = \boxed{3}$$

(1) $f(x) = \frac{1}{x}$, $a = 2$

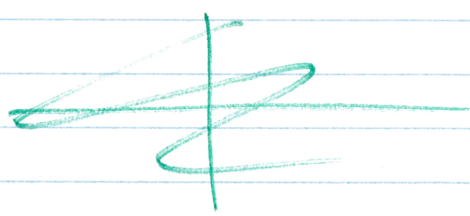
$$\lim_{h \to 0} \frac{\left[\frac{1}{2+h} - \frac{1}{2}\right]}{h} \cdot \frac{2(2+h)}{2(2+h)} = \lim_{h \to 0} \frac{2 - (2+h)}{2h(2+h)}$$

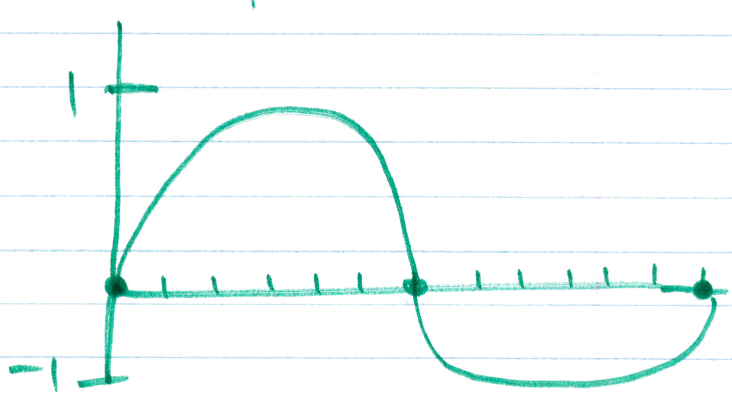
$$\lim_{h \to 0} \frac{\cancel{2} - \cancel{2} - h}{2h(2+h)} = \lim_{h \to 0} \frac{-\cancel{h}}{2\cancel{h}(2+h)} = \lim_{h \to 0} \frac{-1}{2(2+h)} = \boxed{-\frac{1}{4}}$$

(22)



(21) not really





3.1

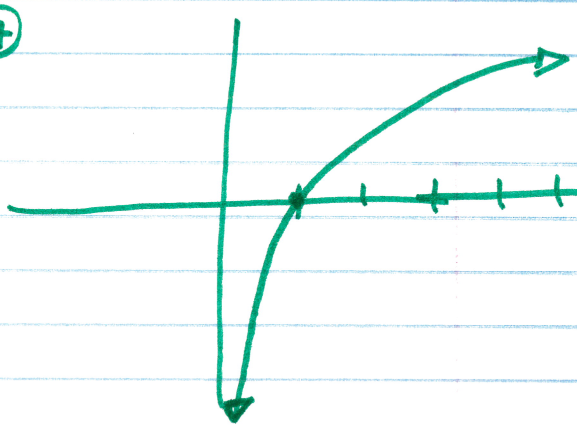
11 $$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$\lim_{h \to 0} 2x + h = \boxed{2x}$$

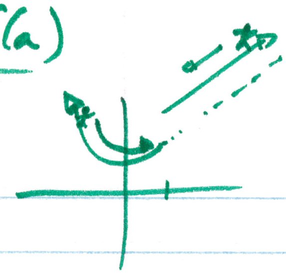
3.1

22

24



$$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$



3.1

42 $$f(x)=\begin{cases} x^2, & x \le 1 \\ 2x, & x > 1 \end{cases}$$

(a) $$\lim_{h\to 0^-} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0^-} \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$\lim_{h\to 0^-} \frac{\cancel{h}(2x+h)}{\cancel{h}} = \lim_{h\to 0^-} 2x+h = 2x$$

(b) $$\lim_{h\to 0^+} \frac{2(x+h) - 2x}{h} = \lim_{h\to 0^+} \frac{\cancel{2x}+2h-\cancel{2x}}{h} = \lim_{h\to 0^+} \frac{2\cancel{h}}{\cancel{h}}$$

$$\lim_{h\to 0^+} 2 = 2$$

$$f'(x)=\begin{cases} 2x, & x \le 1 \\ 2, & x > 1 \end{cases}$$

(c) $\lim_{x\to 1^-} f'(x) = 2$ (d) $\lim_{x\to 1^+} f'(x) = 2$

(e) $\lim_{x\to 1}$ exists because $\lim_{x\to 1^-} = \lim_{x\to 1^+}$

(f) $$\lim_{h\to 0^-} \frac{(1+h)^2 - 1^2}{h} = \lim_{h\to 0^-} \frac{\cancel{1}+2h+h^2-\cancel{1}}{h} = \lim_{h\to 0^-} \frac{\cancel{h}(2+h)}{\cancel{h}}$$

$$\lim_{h\to 0^-} 2+h = 2$$

(g) $$\lim_{h\to 0^+} \frac{2(1+h) - 1^2}{h} = \lim_{h\to 0^+} \frac{2+2h-1}{h} = \lim_{h\to 0^+} \frac{1+2h}{h}$$

$$\lim_{h\to 0^+} \frac{1}{h} + \frac{2\cancel{h}}{\cancel{h}} = \lim_{h\to 0^+} \frac{1}{h} + 2 = \infty$$

(h) $f'(1)$ DOES NOT EXIST BECAUSE THE RIGHT-HAND DERIVATIVE AND

$(1+h)(1+h)(1+h)$

$(1+2h+h^2)(1+h)$

$1+2h+h^2+h+2h^2+h^3$

3.1

44 $f(x) = \begin{cases} x^3 & , x \le 1 \\ 3x+k & , x > 1 \end{cases}$

CONTINUOUS: $1^3 = 3(1)+k$

$1 = 3+k$

$\underline{-2 = k}$

$$\lim_{h \to 0^-} \frac{(1+h)^3 - 1^3}{h} = \frac{\cancel{1}+3h+3h^2+h^3-\cancel{1}}{h}$$

$$\lim_{h \to 0^-} \frac{\cancel{h}(3+3h+h^2)}{\cancel{h}} = \lim_{h \to 0^-} 3+3h+h^2 = 3$$

$$\lim_{h \to 0^+} \frac{3(1+h)+k - 1^3}{h} = \lim_{h \to 0^+} \frac{3+3h+\overset{-2}{k}-1}{h}$$

$$\lim_{h \to 0^+} \frac{3h}{h} = \lim_{h \to 0^+} 3 = 3$$