3.6

5

$$y = \left(\frac{\sin x}{1+\cos x}\right)^2$$

$$y' = 2\left(\frac{\overset{f}{\sin x}}{\underset{g}{1+\cos x}}\right)\left[\frac{\cos x(1+\cos x) - -\sin x \cdot \sin x}{(1+\cos x)^2}\right]$$

$$2\left(\frac{\sin x}{1+\cos x}\right)\left[\frac{\cos x + (\cos^2 x + \sin^2 x)}{(1+\cos x)^2}\right]$$

$$2\left(\frac{\sin x}{\cancel{1+\cos x}}\right)\left[\frac{\cancel{\cos x + 1}}{(1+\cos x)^2}\right] = \frac{2\sin x}{(1+\cos x)^2}$$

33 $f(u) = u^5 + 1 \qquad u = \sqrt{x} = \frac{1}{2}x^{-\frac{1}{2}}$

$x^{1/2}$

$f'(u) = 5u^4\, du$

$$f'(x) = 5(\sqrt{x})^4 \cdot \frac{1}{2}x^{-1/2} \qquad f(x) = (\sqrt{x})^5 + 1$$

$$= \frac{5}{2}x^2 \cdot x^{-1/2} \qquad = x^{5/2} + 1$$

$$= \frac{5}{2}x^{\frac{3}{2}}$$

$$f'(1) = \frac{5}{2}(1)^{\frac{3}{2}} = \frac{5}{2}$$

37 $f(u) = \dfrac{\overset{f}{2u}}{\underset{g}{u^2+1}} \qquad u = 10x^2 + x + 1 \qquad du = 20x + 1$

$$f'(u) = \frac{(2du)(u^2+1) - (2u\,du)(2u)}{(u^2+1)^2}$$

$$= \frac{2(20x+1)((10x^2+x+1)^2+1) - 2(10x^2+x+1)^2(20x+1)\cdot 2}{((10x^2+x+1)^2+1)^2}$$

3.6

15. $y = \sin^{-5} x - \cos^3 x$

$y = (\sin x)^{-5} - (\cos x)^3$

$y' = -5(\sin x)^{-6}[\cos x] - 3(\cos x)^2[-\sin x]$

$= \dfrac{-5\cos x}{\sin^6 x} + 3 \sin x \cos^2 x$

$-5 \cos x \csc^6 x$

31. $y = \cot(3x-1)$

$y' = -\csc^2(3x-1)\,[3]$

$= -3\csc^2(3x-1) \qquad -3\left(\csc(3x-1)\right)^2$

$y'' = -6\left(\csc(3x-1)\right)[-\csc(3x-1)\cot(3x-1)][3]$

$= +18\left(\csc^2(3x-1)\right)[\cot(3x-1]$

53. $y = \sin\left(\frac{1}{2}x\right)$

$y' = \cos\left(\frac{1}{2}x\right)\left[\frac{1}{2}\right]$

17. $y = \underset{f}{(\sin x)^3}\ \underset{g}{\tan 4x}$

$y' = 3(\sin x)^2[\cos x]\tan 4x + \sec^2 4x\,[4]\sin^3 x$

$= 3\sin^2 x \cos x \tan 4x + 4\sec^2 4x \sin^3 x$

$[\cos(3x-2)[3]]$

3.6

(21) $y=(\sin(3x-2))^2$

$y = 2(\sin(3x-2))[\cos(3x-2)][3]$

$= 6\sin(3x-2)\cos(3x-2)$

$30^\circ \cdot \frac{\pi}{180^\circ}$

(57) $\frac{d}{dx}\cos(x^\circ)$

$\frac{d}{dx}\cos\left(\frac{\pi}{180}x\right)$

$-\sin\left(\frac{\pi}{180}x\right)\left[\frac{\pi}{180}\right]$

$-\frac{\pi}{180}\sin(x^\circ)$

$(\cos 7x)^2$

(23) $y=(1+\cos^2 7x)^3$

$y' = 3(1+\cos^2 7x)^2[2(\cos 7x)(-\sin 7x)[7]]$

$y' = -42(1+\cos^2 7x)(\cos 7x)(\sin 7x)$

3.6

(13) $y = (x + \sqrt{x})^{-2}$

$y = (x + x^{1/2})^{-2}$

$y' = -2(x + x^{1/2})^{-3} [1 + \frac{1}{2}x^{-\frac{1}{2}}]$ ✓

$y' = (x + x^{1/2})^{-3} [-2 - x^{-\frac{1}{2}}]$ ✓

$y' = \dfrac{-2 - x^{-\frac{1}{2}}}{(x + x^{1/2})^3}$ ✓

$x^{-\frac{1}{2}}$

$\dfrac{1}{\sqrt{x}}$

(23) $y = (1 + \cos^2 7x)^3$

$y = (1 + (\cos 7x)^2)^3$

$y' = 3(1 + (\cos 7x)^2)^2 [2(\cos 7x)] [-\sin 7x] [7]$

$y' = -42(1 + (\cos 7x)^2)^2 (\cos 7x)(\sin 7x)$

(33) $f(u) = u^5 + 1$ $\quad u = g(x) = \sqrt{x} = x^{1/2}$ $\quad x = 1$

$f'(u) = 5u^4 du$ $\quad du = \frac{1}{2}x^{-\frac{1}{2}}$

$f'(x) = 5(x^{1/2})^4 [\frac{1}{2}x^{-\frac{1}{2}}]$

$f'(1) = 5(1^{1/2})^4 [\frac{1}{2}(1)^{-\frac{1}{2}}] = \frac{5}{2}$

(19) $y = \dfrac{3}{\sqrt{2x+1}} = 3(2x+1)^{-\frac{1}{2}}$

$y' = -\frac{3}{2}(2x+1)^{-\frac{3}{2}} [2]$

$= \underline{-3(2x+1)^{-\frac{3}{2}}} = \underline{\dfrac{-3}{\sqrt{(2x+1)^3}}}$

3.6 $\qquad f'g+g'f$

(26) $r = \overbrace{\sec 2\theta}^{f}\ \overbrace{\tan 2\theta}^{g}$

$r' = \overbrace{\sec 2\theta \tan 2\theta\,[2]}^{f'}\ \overset{g}{\tan 2\theta} + \sec^2 2\theta\ [2]\ \sec 2\theta$

$= 2\sec 2\theta \tan^2 2\theta + 2\sec^3 2\theta$

(53) $y = \sin(x/2) = \sin(\tfrac{1}{2}x)$ $\qquad 2x^2$

$y' = \cos(\tfrac{1}{2}x)\,[\tfrac{1}{2}]$ $\qquad 2(2x)$

(56) (a) $2f(x)$

$2f'(x)$

$2\underline{f'(2)}$

$2(\tfrac{1}{3}) = \underline{\tfrac{2}{3}}$

(b) $f(x)+g(x)$

$f'(x)+g'(x)$

$f'(3)+g'(3)$

$\boxed{2\pi + 5}$

(c) $f(x)\cdot g(x)$

$f'(x)\cdot g(x) + g'(x)\cdot f(x)$

$f'(3)\cdot g(3) + g'(3)\cdot f(3)$

$(2\pi)(-4)+(5)(3)$

$\boxed{-8\pi + 15}$

(d) $f(x)/g(x) = \dfrac{f(x)}{g(x)}$

$\dfrac{f'(x)\cdot g(x) - g'(x)\cdot f(x)}{(g(x))^2}$

$\dfrac{(\tfrac{1}{3})(2) - (-3)(8)}{2^2}$

$\dfrac{(\tfrac{2}{3} + 24)\,3}{(4)\,3}$

$\dfrac{2+72}{12}$

$\dfrac{74}{12} = \dfrac{37}{6}$

(e) $f(g(x))$

$f'(g(x))\,[g'(x)]$

$f'(\overset{2}{g(2)})\,[g'(2)]$

$\tfrac{1}{3} \cdot -3 = -1$

(f) $\sqrt{f(x)} = (f(x))^{1/2}$

$\tfrac{1}{2}(f(x))^{-\frac{1}{2}}\,[f'(x)]$

$\tfrac{1}{2}(8)^{-\frac{1}{2}}\ (\tfrac{1}{3})$

$\dfrac{1}{2\sqrt{8}} \cdot \dfrac{1}{3}$

$\dfrac{1}{2\cdot 2\sqrt{2}} \cdot \dfrac{1}{3} = \dfrac{1}{12\sqrt{2}}$

3.6

(g) $\frac{1}{g^2(x)} = (g(x))^{-2}$

$-2(g(x))^{-3}[g'(x)]$

$-2(-4)^{-3}[5]$

$-10(-4)^{-3} = \frac{-10}{-64} = \frac{5}{32}$

(h) $\sqrt{f^2(x)+g^2(x)}$

$(f^2(x)+g^2(x))^{1/2}$

$\frac{1}{2}(\quad)^{-1/2}[2f(x)f'(x)$
$+2g(x)g'(x)]$

3.6 $\quad f$

(5) $y = \left(\dfrac{\sin x}{1+\cos x}\right)^2$ $\quad g$

$$\frac{dy}{dx} = 2\left(\frac{\sin x}{1+\cos x}\right)\left[\frac{\cos x\,(1+\cos x) - -\sin x \cdot \sin x}{(1+\cos x)^2}\right]$$

$$= 2\left(\frac{\sin x}{1+\cos x}\right)\left[\frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}\right]$$

$$= 2\left(\frac{\sin x}{\cancel{1+\cos x}}\right)\left[\frac{\cancel{\cos x + 1}}{(1+\cos x)^2}\right]$$

7x

7

$$= \frac{2\sin x}{(1+\cos x)^2}$$

(35) $f(u) = \cot \frac{\pi}{10} u$ $\qquad u = 5\sqrt{x} = 5x^{1/2} \quad du = \frac{5}{2}x^{-1/2}$

$$f'(u) = -\csc^2 \frac{\pi}{10} u \left[\frac{\pi}{10}\, du\right]$$

$$f'(x) = -\csc^2\left(\frac{\pi}{\cancel{10}_{2}} \cdot \cancel{5}\sqrt{x}\right)\left[\frac{\pi}{\cancel{10}_{2}} \cdot \frac{\cancel{5}}{2} x^{-1/2}\right]$$

$$= -\csc^2\left(\frac{\pi}{2}\sqrt{x}\right)\left[\frac{\pi}{4} x^{-1/2}\right] \rightarrow \frac{\pi}{4} - \csc^2\left(\frac{\pi}{2}\sqrt{x}\right)\frac{1}{x^{1/2}}$$

$$= \frac{-\pi \csc^2\left(\frac{\pi}{2}\sqrt{x}\right)}{4\sqrt{x}}$$