3.7

25 $y = 2\sin(\pi x - y)$ @ (1,0)

$$\frac{dy}{dx} = \underline{2\cos(\pi x - y)}\left[\pi - \frac{dy}{dx}\right]$$

$$\frac{dy}{dx} = 2\pi\cos(\pi x - y) - \cancel{2\cos(\pi x - y)\frac{dy}{dx}}$$

$$+ 2\cos(\pi x - y)\frac{dy}{dx} \qquad + \cancel{2\cos(\pi x - y)\frac{dy}{dx}}$$

$$\frac{\frac{dy}{dx}\left[1 + 2\cos(\pi x - y)\right]}{1 + 2\cos(\pi x - y)} = \frac{2\pi\cos(\pi x - y)}{1 + 2\cos(\pi x - y)} = \frac{dy}{dx}$$

$$y - 0 = 2\pi(x - 1) \qquad = \frac{2\pi\cos(\pi \cdot 1 - 0)}{1 + 2\cos(\pi \cdot 1 - 0)}$$

(a) $\boxed{y = 2\pi(x-1)}$

$$= \frac{-2\pi}{-1} = 2\pi$$

(b) $y - 0 = \frac{-1}{2\pi}(x-1)$

$$\boxed{y = \frac{-1}{2\pi}(x-1)}$$

x^{200}

x^2

x^5 $\quad 3x^5$ $\quad (\ln \sin(x^2))^2$

$4x^3$ $\quad 15x^4$ $\quad (3x+1)^{200}$

3.7

27 $x^2+y^2=1$

$2x+2y\frac{dy}{dx}=0$

$-2x \qquad -2x$

$\frac{\cancel{2y}\frac{dy}{dx}}{\cancel{2y}}=\frac{-2x}{2y}$

$\frac{dy}{dx}=\frac{-x}{y}\begin{matrix}f\\g\end{matrix}$

$\frac{d^2y}{dx^2}=\frac{-y++x\frac{dy}{dx}}{y^2}$

$=\frac{\left[-y+x\left(-\frac{x}{y}\right)\right]y}{y^2\cdot y}$

$=\frac{-y^2-x^2}{y^3}$

$\frac{1}{\sqrt{2}} \qquad \frac{2}{2}=1$

$\frac{(x^2-y^2)}{\left(\frac{y^2}{x}\right)}\frac{x}{x}$

43 $f''(x)=x^{-1/3}$

(b) $f(x)=\frac{9}{10}x^{5/3}-7$

$f'(x)=\frac{3}{2}x^{2/3}$

✓ $f''(x)=x^{-1/3}$

(c) $f'''(x)=-\frac{1}{3}x^{-4/3}$

$f''(x)=x^{-1/3}$

✓ $f'''(x)=-\frac{1}{3}x^{-4/3}$

~~(a)~~ $f(x)=\frac{3}{2}x^{2/3}-3$

$f'(x)=x^{-1/3}$

$f''(x)=$ ✗

(d) $f'(x)=\frac{3}{2}x^{2/3}+6$

✓ $f''(x)=x^{-1/3}$

3.7

(5) $x = \tan y$

$$\frac{1}{\sec^2 y} = \frac{\sec^2 y \frac{dy}{dx}}{\sec^2 y}$$

$$\cos^2 y = \frac{dy}{dx}$$

$(\tan y)^2$

$2 \tan y [\sec^2 y][\frac{dy}{dx}]$

(37) $y = x\sqrt{x^2+1} = \overset{f}{x}\,\overset{g}{(x^2+1)^{1/2}}$

$$\frac{dy}{dx} = (x^2+1)^{1/2} + \cancel{\tfrac{1}{2}}(x^2+1)^{-1/2}[\cancel{2}x]\cdot x \rightarrow \frac{x^2}{(x^2+1)^{1/2}}$$

$$= \sqrt{x^2+1} + \frac{x^2}{\sqrt{x^2+1}}$$

(53) $v = \underline{8(s-t)^{1/2}} + 1$

$v' \frac{d^2s}{dt^2}$ a

$$\frac{dv}{dt} = 4(s-t)^{-1/2}\left[\frac{ds}{dt} - 1\right]$$

$$= 4(s-t)^{-1/2}[8(s-t)^{1/2} \cancel{+1-1}]$$

$$= \frac{32\cancel{(s-t)^{1/2}}}{\cancel{(s-t)^{1/2}}}$$

$$= 32$$

$2(1+3)(4+5)$

$(2+6)(8+10)$

3.7

(22) $x^2 - \overset{f\,g}{\sqrt{3}\,xy} + 2y^2 = 5$

$$2x - \sqrt{3}\left(y + x\frac{dy}{dx}\right) + 4y\frac{dy}{dx} = 0$$

$$2x - \sqrt{3}y - \sqrt{3}x\frac{dy}{dx} + 4y\frac{dy}{dx} = 0$$

$$-2x + \sqrt{3}y \qquad\qquad -2x + \sqrt{3}y$$

$$\frac{\frac{dy}{dx}(-\sqrt{3}x + 4y)}{-\sqrt{3}x + 4y} = \boxed{\frac{\overset{f}{-2x + \sqrt{3}y}}{\underset{g}{-\sqrt{3}x + 4y}} = \frac{dy}{dx}}$$

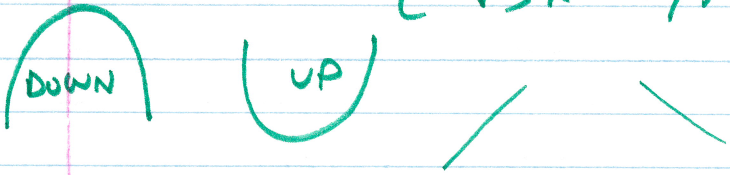
$$\frac{\left(-2 + \sqrt{3}\frac{dy}{dx}\right)(-\sqrt{3}x + 4y) - \left(-\sqrt{3} + 4\frac{dy}{dx}\right)(-2x + \sqrt{3}y)}{(-\sqrt{3}x + 4y)^2}$$

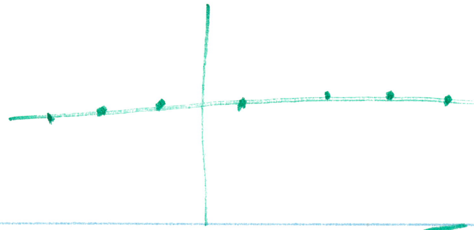
$-\sqrt{3}x+4y$

$$\left[\frac{\left(-2 + \sqrt{3}\left(\frac{-2x + \sqrt{3}y}{-\sqrt{3}x + 4y}\right)\right)(-\sqrt{3}x + 4y) - \left(-\sqrt{3} + 4\left(\frac{-2x + \sqrt{3}y}{-\sqrt{3}x + 4y}\right)\right)(-2x + \sqrt{3}y)}{(-\sqrt{3}x + 4y)^2}\right.$$

$-[\sqrt{3}x + 4y]$

$$\frac{\left(2\sqrt{3}x - 8y + \sqrt{3}(-2x + \sqrt{3}y)\right)(-\sqrt{3}x + 4y) - \left(3x - 4\sqrt{3}y + 4(-2x + \sqrt{3}y)\right)(-2x + \sqrt{3}y)}{(-\sqrt{3}x + 4y)^3}$$





3.7

(49) $x^2 + xy + y^2 = 7$ $\qquad (\sqrt{7}, 0) \qquad (-\sqrt{7}, 0)$

$x^2 + x(0) + (0)^2 = 7$

$x = \pm\sqrt{7}$

$$\overset{f\ g}{x^2 + xy} + y^2 = 7$$

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$-2x \quad -y \qquad\qquad -2x - y$$

$$\frac{dy}{dx}(\cancel{x+2y}) = \frac{-2x - y}{x + 2y}$$

$$\frac{-2(\sqrt{7}) - 0}{\sqrt{7} + 2(0)} = \frac{-2\cancel{\sqrt{7}}}{\cancel{\sqrt{7}}} = -2$$

$$\frac{-2(-\sqrt{7}) - 0}{-\sqrt{7} + 2(0)} = \frac{2\cancel{\sqrt{7}}}{-\cancel{\sqrt{7}}} = -2$$

(4) $x^2 = \dfrac{x - y}{x + y}$ $\quad f,\ g$

$$(x+y)^2 \quad 2x = \frac{(1 - \frac{dy}{dx})(x+y) - (1 + \frac{dy}{dx})(x-y)}{(x+y)^2} \cdot (x+y)^2$$

$$(x+y)^2 \cdot 2x = \cancel{x} + y - x\frac{dy}{dx} \cancel{- y\frac{dy}{dx}} \cancel{- x} + y - x\frac{dy}{dx} \cancel{+ y\frac{dy}{dx}}$$

$$(x+y)^2 \cdot 2x = 2y - 2x\frac{dy}{dx}$$

$$-2y \qquad -2y$$

$$\frac{2x(x+y)^2 - 2y}{-2x} = \frac{\cancel{-2x}\frac{dy}{dx}}{\cancel{-2x}}$$

$$-\frac{x(x+y)^2 - y}{x} = \frac{dy}{dx}$$

3.7

5 $x = \tan y$

$$\frac{1}{\sec^2 y} = \frac{\cancel{\sec^2 y}\,\frac{dy}{dx}}{\cancel{\sec^2 y}}$$

$$\cos^2 y = \frac{dy}{dx}$$

19 $\overset{f}{x^2}\overset{g}{y^2} = 9$, $(-1,3)$

$\cancel{2xy^2} + 2x^2y\frac{dy}{dx} = 0$
$\cancel{-2xy^2}$ $\qquad -2xy^2$

$$\frac{\cancel{2x^2y}\frac{dy}{dx}}{\cancel{2x^2y}} = \frac{-\cancel{2}xy^2}{\cancel{2}x^2y}$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

$$\frac{dy}{dx}(-1,3) = \frac{-3}{-1} = 3$$

(a) TANGENT LINE

$y - 3 = 3(x+1)$

(b) NORMAL LINE

$y - 3 = -\frac{1}{3}(x+1)$

21 $6x^2 + \overset{f\ g}{3xy} + 2y^2 + 17y - 6 = 0$ $\qquad(-1,0)$

$12x + 3y + 3x\frac{dy}{dx} + 4y\frac{dy}{dx} + 17\frac{dy}{dx} = 0$
$-12x \quad -3y \qquad\qquad -12x - 3y$

$$\frac{\frac{dy}{dx}\cancel{(3x+4y+17)}}{\cancel{(\ )}} = \frac{-12x-3y}{3x+4y+17} = \frac{dy}{dx}$$

$$\frac{-12(-1)-3(0)}{3(-1)+4(0)+17} = \frac{dy}{dx}(-1,0)$$

$$\frac{12}{14} =$$

$$\frac{6}{7} =$$

TANGENT:

$y = \frac{6}{7}(x+1)$

NORMAL

$y = -\frac{7}{6}(x+1)$

3.7

(23) $2xy + \pi \sin y = 2\pi \qquad (1, \frac{\pi}{2})$

$\cancel{2y} + 2x\frac{dy}{dx} + \pi\cos y \frac{dy}{dx} = 0$

$-\cancel{2y} \qquad\qquad\qquad -2y$

$\frac{\frac{dy}{dx}(2x + \cancel{\pi\cos y})}{(\cancel{\ })} = \frac{-2y}{2x+\pi\cos y} = \frac{dy}{dx}$

$= \frac{-2(\frac{\pi}{2})}{2(1)+\pi\cos\frac{\pi}{2}} = \frac{dy}{dx}(1, \frac{\pi}{2})$

$= \frac{-\pi}{2+0} = \frac{-\pi}{2}$

TANGENT LINE:

$y - \frac{\pi}{2} = -\frac{\pi}{2}(x-1)$

NORMAL LINE:

$y - \frac{\pi}{2} = \frac{2}{\pi}(x-1)$

(29) $y^2 = x^2 + 2x$

$\frac{2y\frac{dy}{dx}}{2y} = \frac{2x+2}{2y}$

$\frac{dy}{dx} = \frac{x+1}{y} \quad \frac{f}{g}$

$\frac{d^2y}{dx^2} = \frac{y - (x+1)\frac{dy}{dx}}{y^2}$

$= \frac{y - (x+1)\frac{(x+1)}{y}}{y^2} \cdot \frac{y}{y}$

$= \frac{y^2 - (x+1)^2}{y^3}$

3.7

37 $y = x\sqrt{x^2+1}$

$$y = \overset{f}{x}\,\overset{g}{(x^2+1)^{1/2}}$$

$$\frac{dy}{dx} = (x^2+1)^{1/2} + \cancel{\tfrac{1}{2}}(x^2+1)^{-1/2}[\cancel{2}x]\cdot x$$

$$= (x^2+1)^{1/2} + x^2(x^2+1)^{-1/2}$$

$$\frac{1}{\sqrt{5}}$$

47b $\overset{f}{x^3}\overset{g}{y^2} = \cos(\pi y)$ $(-1,1)$

$$3x^2y^2 + 2x^3y\frac{dy}{dx} = -\sin(\pi y)\left[\pi\frac{dy}{dx}\right]$$

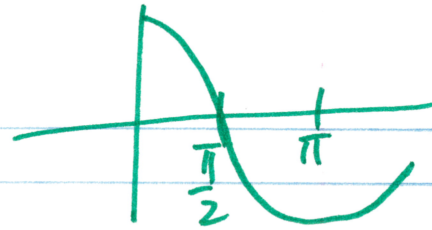
$$3(-1)^2(1)^2 + 2(-1)^3(1)\frac{dy}{dx} = -\sin((\pi)(1))\left[\pi\frac{dy}{dx}\right]$$

$$3 - 2\frac{dy}{dx} = 0$$

$$\frac{3}{2} = \frac{2\frac{dy}{dx}}{2}$$

$$\frac{3}{2} = \frac{dy}{dx}$$

3.7



(25) $y = 2\sin(\pi x - y)$ $\quad (1,0)$

$$\frac{dy}{dx} = 2\cos(\pi x - y)\left[\pi - \frac{dy}{dx}\right]$$

$$\frac{dy}{dx} = 2\pi\cos(\pi x - y) - 2\cos(\pi x - y)\frac{dy}{dx}$$

$$+2\cos(\pi x - y)\frac{dy}{dx} \qquad\qquad +2\cos(\pi x - y)\frac{dy}{dx}$$

$$\frac{\frac{dy}{dx}\left(2\cos(\pi x - y) + 1\right)}{\left(2\cos(\pi x - y)+1\right)} = \boxed{\frac{2\pi\cos(\pi x - y)}{2\cos(\pi x - y)+1} = \frac{dy}{dx}}$$

$$\boxed{y = 2\pi(x-1)} \text{ TANGENT} \quad = \frac{2\pi\cos(\pi(1)-0)}{2\cos(\pi(1)-0)+1} = \frac{dy}{dx}(1,0)$$

$$\boxed{y = -\frac{1}{2\pi}(x-1)} \text{ NORMAL} \quad = \frac{-2\pi}{-1} = 2\pi$$

(28) $x^{2/3} + y^{2/3} = 1$

$$\cancel{\tfrac{2}{3}x^{-1/3}} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

$$-\frac{2}{3}x^{-1/3} \qquad\qquad -\frac{2}{3}x^{-1/3}$$

$$\frac{\cancel{\frac{2}{3}y^{-1/3}}\frac{dy}{dx}}{\cancel{\frac{2}{3}y^{-1/3}}} = \frac{-\cancel{\frac{2}{3}}x^{-1/3}}{\cancel{\frac{2}{3}}y^{-1/3}}$$

$$\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}} \quad \begin{matrix} f \\ g \end{matrix}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{1}{3}y^{-2/3}x^{1/3}\frac{dy}{dx} - \frac{1}{3}x^{-2/3}y^{1/3}}{\left(x^{1/3}\right)^2}$$

$$= \frac{-\frac{1}{3}y^{-2/3}\cancel{x^{1/3}}\left(\frac{-y^{1/3}}{\cancel{x^{1/3}}}\right) + \frac{1}{3}x^{-2/3}y^{1/3}}{x^{2/3}}$$

$$= \frac{\left(\frac{1}{3}y^{-1/3} + \frac{1}{3}x^{-2/3}y^{1/3}\right)}{x^{2/3}} \cdot \frac{y^{1/3}x^{2/3}}{y^{1/3}x^{2/3}}$$

$$= \frac{1}{3}\left(\boxed{\frac{x^{2/3} + y^{2/3}}{3x^{4/3}y^{1/3}}}\right)$$