3.8

(7) $y = x\sin^{-1}x + \sqrt{1-x^2}$

$y = \overset{f}{x}\,\overset{g}{\sin^{-1}x} + (1-x^2)^{1/2}$

$y' = \sin^{-1}x + \overset{g'}{\frac{1}{\sqrt{1-x^2}}}\cdot \overset{f}{x} + \cancel{\frac{1}{2}}(1-x^2)^{-\frac{1}{2}}\left[-\cancel{2}x\right]$

$= \sin^{-1}x + \cancel{\frac{x}{\sqrt{1-x^2}}} - \cancel{\frac{x}{\sqrt{1-x^2}}}$

$= \sin^{-1}x$

(21) $y = \tan^{-1}\overset{(x^2-1)^{1/2}}{\sqrt{x^2-1}} + \csc^{-1}x$

$y' = \frac{1}{(\sqrt{x^2-1})^2+1}\left[\cancel{\frac{1}{2}}(x^2-1)^{-1/2}\right]\left[\cancel{2}x\right] + \frac{-1}{|x|\sqrt{x^2-1}}$

$= \frac{1}{x^2\cancel{-1}\cancel{+1}}\cdot\frac{x}{\sqrt{x^2-1}} + \frac{-1}{|x|\sqrt{x^2-1}}$

$= \frac{\cancel{x}}{x^{2}\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}}$

$= \frac{1}{x\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}} = 0$

3.8

$(1-s^2)^{1/2}$

(6) $y = s\sqrt{1-s^2} + \cos^{-1} s$

$y' = \sqrt{1-s^2} + \cancel{\tfrac{1}{2}}(1-s^2)^{-1/2}[-\cancel{2}s] \cdot s + \dfrac{-1}{\sqrt{1-s^2}}$

$\dfrac{\sqrt{1-s^2}}{\sqrt{1-s^2}}$

$= \sqrt{1-s^2} - \dfrac{s^2}{\sqrt{1-s^2}} - \dfrac{1}{\sqrt{1-s^2}}$

$= \dfrac{\cancel{1} - s^2 - s^2 - \cancel{1}}{\sqrt{1-s^2}}$ $\boxed{= \dfrac{-2s^2}{\sqrt{1-s^2}} = y'}$

(31) $x = \arctan t$

(a) $x' = \dfrac{1}{1+t^2}$ $\qquad \dfrac{1}{1+t^2} > 0$

$= (1+t^2)^{-1}$

(b) $x'' = -((1+t^2)^{-2}[2t]$

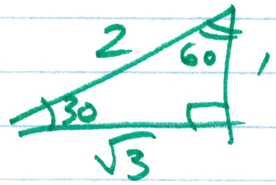
$= \dfrac{-2t}{(1+t^2)^2}$ $\qquad t \geq 0$

3.8

(23) $y=\sec^{-1}x \qquad x=\left(2,\frac{\pi}{3}\right)$

$y=\sec^{-1}2 \quad \frac{2}{1}$ hyp adj

$y=\frac{\pi}{3}$



$\sec\frac{\pi}{3}=2$

$\sec^{-1}(2)=\frac{\pi}{3}$

$$y'=\frac{1}{|x|\sqrt{x^2-1}}=\frac{1}{|2|\sqrt{2^2-1}}=\frac{1}{2\sqrt{3}}$$

$$\boxed{y-\frac{\pi}{3}=\frac{1}{2\sqrt{3}}(x-2)}$$

(9) $x(t)=\sin^{-1}\left(\frac{t}{4}\right) \qquad \frac{1}{4}t$

$$x'(t)=\frac{1}{\sqrt{1-\left(\frac{t}{4}\right)^2}}\left[\frac{1}{4}\right]$$

$$=\frac{1}{\sqrt{1-\frac{t^2}{16}}}\left[\frac{1}{4}\right]$$

$$=\frac{1}{\sqrt{\frac{1}{16}(16-t^2)}}\left[\frac{1}{4}\right]$$

$$=\frac{1}{\sqrt{\frac{1}{16}}\sqrt{16-t^2}}$$

$$=\frac{1}{\cancel{\frac{1}{4}}\sqrt{16-t^2}}\cancel{\left[\frac{1}{4}\right]}$$

$$=\frac{1}{\sqrt{16-t^2}}$$

$\sqrt{8}$

$\sqrt{4\cdot 2}$

$2\sqrt{2}$

$(xy)^2=x^2y^2$

$x'(3)=\frac{1}{\sqrt{16-3^2}}=\frac{1}{\sqrt{7}}$

3.8 $\quad t^{-1} \quad -t^{-2}$

17. $y = \sec^{-1} \frac{1}{t}$

$$y' = \frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2 - 1}} \left[-\frac{1}{t^2}\right]$$

$$= \frac{1}{\cancel{\left|\frac{1}{t}\right|}\sqrt{\frac{1}{t^2} - 1}} \cancel{\left[-\frac{1}{t^2}\right]} \quad -\frac{1}{t}$$

$$= \frac{1}{\sqrt{\frac{1}{t^2}(1 - t^2)}} \cdot -\frac{1}{t}$$

$$\frac{1}{\sqrt{\frac{1}{t^2}}\sqrt{1-t^2}} \cdot -\frac{1}{t}$$

$$\frac{1}{\cancel{\frac{1}{t}}\sqrt{1-t^2}} \cdot -\cancel{\frac{1}{t}} = \frac{-1}{\sqrt{1-t^2}}$$

3.8

(29) $f(x) = \cos x + 3x$ $\quad f'(x) = -\sin x + 3$

(a) $f(0) = \cos 0 + 3(0) = 1$ $\quad f'(0) = -\sin 0 + 3 = 3$

$f(x) \longleftrightarrow f^{-1}(x)$

$(0,1) \longleftrightarrow (1,0)$

(b) $f^{-1}(1) = 0$ $\quad (f^{-1})'(1) = \frac{1}{3}$

$f(0) = 1$ $\quad \underline{f'(0) = 3}$ $\quad$ suppose $g(x) = f^{-1}(x)$

$\underline{g(1)} = 0$

$g'(1) = \frac{1}{3}$

AP TEST

$f(2) = 3$ $\quad f'(2) = 2$ $\quad g(x) = f^{-1}(x)$

$g'(3) = \frac{1}{2}$

$y = a^x$

$y' = a^x \ln a \frac{dy}{dx}$

$y = \underline{5}^{\sin x}$

$y' = \underline{5}^{\sin x} \ln 5 [\cos x]$

$y = \ln x = \frac{1}{x} \frac{dy}{dx}$

$\log_a x = y$

$a^y = x$

$y = \ln(\sin x)$

$y' = \frac{1}{\sin x}[\cos x] = \cot x$

$y = \log_3(\sin x)$

$y' = \frac{1}{\ln 3 \sin x}[\cos x]$

$\frac{1}{4}x$

3.8

(25) $y = \sin^{-1}\left(\frac{x}{4}\right)$; $x = 3$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{x}{4}\right)^2}}\left[\frac{1}{4}\right]$

$y = \sin^{-1}\left(\frac{3}{4}\right)$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{3}{4}\right)^2}}\left[\frac{1}{4}\right]$

$y = .848$

$= \frac{1}{\sqrt{7}}$ $\approx$ $.377$

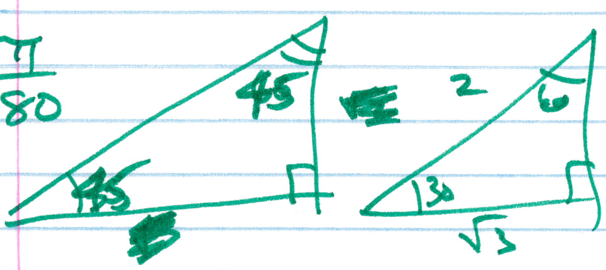
$(3, .848)$

$\frac{1}{\sqrt{1-\frac{9}{16}}}\left[\frac{1}{4}\right]$

$$\boxed{y - .848 = \frac{1}{\sqrt{7}}(x-3)}$$

$\frac{1}{\sqrt{\frac{7}{16}}}\left[\frac{1}{4}\right]$

$60 \cdot \frac{\pi}{180}$



$\sqrt{\frac{16}{7}}\left[\frac{1}{4}\right]$

$\frac{\cancel{4}}{\sqrt{7}} \cdot \frac{1}{\cancel{4}} = \frac{1}{\sqrt{7}} = m$

(23) $y = \sec^{-1} x$; $x = 2$

$y' = \frac{1}{|x|\sqrt{x^2-1}}$

$\cos^{-1} \frac{1}{2} \frac{\text{adj}}{\text{hyp}}$

$y = \sec^{-1} 2$

$y = \frac{\pi}{3}$

$y' = \frac{1}{2\sqrt{2^2-1}}$

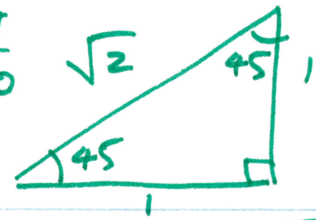
$\left(2, \frac{\pi}{3}\right)$

$$\boxed{y - \frac{\pi}{3} = \frac{1}{2\sqrt{3}}(x-2)}$$

$= \frac{1}{2\sqrt{3}} = m$

$\frac{12 \text{ in}}{1 \text{ ft}}$

$\cancel{45} \cdot \frac{\pi}{\cancel{180}}$ 4



$\frac{\sqrt{2}}{2}$ $\frac{1}{\sqrt{2}}$

$\frac{2}{\sqrt{2}}$ $\sqrt{2}$

3.8

27 (a) $y = \tan x$; $\left(\frac{\pi}{4}, 1\right)$

$$\boxed{y - 1 = 2\left(x - \frac{\pi}{4}\right)}$$

$$y' = \sec^2 x$$

$$y'\left(\frac{\pi}{4}\right) = \sec^2\left(\frac{\pi}{4}\right) = \left(\sec\frac{\pi}{4}\right)^2$$

$$= (\sqrt{2})^2 = 2 = m$$

(b) $y = \tan^{-1} x$ $\left(1, \frac{\pi}{4}\right)$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\boxed{y - \frac{\pi}{4} = \frac{1}{2}(x-1)}$$

$$\frac{dy}{dx}(1) = \frac{1}{1+1^2} = \frac{1}{2}$$

$$\left(\frac{a}{b}\right)^n \qquad \frac{a^n}{b^n}$$

5 $y = \sin^{-1}\frac{3}{t^2}$ $3t^{-2}$ $-6t^{-3}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}}\left[\frac{-6}{t^3}\right]$$

$$= \frac{1}{\sqrt{1-\frac{9}{t^4}}}\left[-\frac{6}{t^3}\right]$$

$$\cancel{\frac{1}{\sqrt{t^4(}}} \qquad \frac{1}{\sqrt{\frac{1}{t^4}(t^4-9)}}\left[-\frac{6}{t^3}\right]$$

$$\frac{1}{\frac{1}{t^2}\sqrt{t^4-9}}\left[\frac{-6}{t^3}\right] = \frac{\cancel{t^2}}{\sqrt{t^4-9}}\left[\frac{-6}{\cancel{t^3}_{\,t}}\right] = \boxed{\frac{-6}{t\sqrt{t^4-9}}}$$

3.8

7) $y = x\sin^{-1}x + \sqrt{1-x^2}$

$y = \overset{f}{x}\,\overset{g}{\sin^{-1}x} + (1-x^2)^{1/2}$

$y' = \sin^{-1}x + \frac{1}{\sqrt{1-x^2}} \cdot x + \cancel{\tfrac{1}{2}}(1-x^2)^{-1/2}[-\cancel{2}x]$

$= \sin^{-1}x + \cancel{\frac{x}{\sqrt{1-x^2}}} - \cancel{\frac{x}{\sqrt{1-x^2}}}$

$\frac{1}{2}x$

$\boxed{y' = \sin^{-1}x}$

37) $\frac{d}{dx}\sin^{-1}\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\left[\frac{1}{2}\right]$

$= \frac{1}{\sqrt{1-\frac{x^2}{4}}}\left[\frac{1}{2}\right]$

$= \frac{1}{\sqrt{\frac{1}{4}(4-x^2)}}\left[\frac{1}{2}\right]$

$= \frac{1}{\cancel{\frac{1}{2}}\sqrt{4-x^2}}\left[\cancel{\frac{1}{2}}\right]$

$= \frac{1}{\sqrt{4-x^2}}$ E.

3.8

13 $y = \sec^{-1}(2s+1)$

$$\frac{dy}{dx} = \frac{1}{|2s+1|\sqrt{(2s+1)^2-1}} \quad [2]$$

$$= \frac{1}{|2s+1|\sqrt{4s^2+4s+\cancel{1}-\cancel{1}}} \quad [2]$$

$$\frac{1}{|2s+1|\sqrt{4(s^2+s)}} \quad [2]$$

$$\frac{1}{|2s+1|\,\cancel{2}\sqrt{s^2+s}} \quad [\cancel{2}]$$

$$\frac{1}{|2s+1|\sqrt{s^2+s}}$$