3.9

(31) $y = \ln(2x)$

$y' = \frac{1}{2x}[2] = \boxed{\frac{1}{x}}$

$y = 1$

$1 = \ln(2x)$

$e^1 = e^{\ln(2x)}$

$e = 2x$

$\frac{e}{2} = x$

$y = mx + b \quad (0,0)$

$y = mx$

$\boxed{\frac{y}{x} = m}$

$\boxed{m = \frac{2}{e}}$

(32) $y = \ln\left(\frac{1}{3}x\right) \longrightarrow \ln\frac{1}{3} + \ln x$

$y' = \frac{1}{\frac{1}{3}x}\left[\frac{1}{3}\right] = \frac{1}{x}$

$y = 1$

$1 = \ln\left(\frac{1}{3}x\right)$

$e = \frac{1}{3}x$

$3e = x$

$y = mx + b \quad (0,0)$

$y = mx$

$\frac{y}{x} = m$

$\boxed{m = \frac{1}{3e}}$

3.9

(51) $P(t) = \frac{300}{1+2^{4-t}} = 300(1+2^{4-t})^{-1}$

(a) $P(0) = 17.5$ (18)

(b) $P'(t) = -300(1+2^{4-t})^{-2}\left[2^{4-t}\ln 2\,[-1]\right]$

$P'(4) = -300(1+2^{4-4})^{-2}\left[-2^{4-4}\ln 2\right]$

$= +300\,2^{-2}\ln 2$

$= \frac{+300\ln 2}{4}$

$= \boxed{75\ln 2}$

(29) $y = 3^x + 1$ $\qquad$ $y = 5x - 1$

$y' = \frac{3^x[\ln 3]}{\ln 3} = \frac{5}{\ln 3}$ $\qquad$ $m = 5$

$3^x = \frac{5}{\ln 3}$ $\qquad$ $\log_a b = \frac{\log b}{\log a} = \frac{\ln b}{\ln a}$

CALCULATOR $\qquad$ PRETTY

$\ln 3^x = \ln\left(\frac{5}{\ln 3}\right)$ $\qquad$ $\log_3 3^x = \log_3\left(\frac{5}{\ln 3}\right)$

$\frac{x\ln 3}{\ln 3} = \frac{\ln\left(\frac{5}{\ln 3}\right)}{\ln 3}$ $\qquad$ $x = \log_3\left(\frac{5}{\ln 3}\right)$

$x =$ $\qquad$ $y = 3^{\log_3\left(\frac{5}{\ln 3}\right)} + 1$ $\qquad$ $\left(\log_3\left(\frac{5}{\ln 3}\right), \frac{5}{\ln 3} + 1\right)$

$y = \frac{5}{\ln 3} + 1$

3.9

(43) $y = (\sin x)^x$

$\ln y = \ln (\sin x)^x$

$\ln y = x \ln (\sin x)$

$y \cdot \frac{1}{y} \frac{dy}{dx} = \left(\ln (\sin x) + \frac{1}{\sin x} [\cos x] \cdot x \right) y$

$= \left(\ln (\sin x) + x \cot x \right) (\sin x)^x$

(45) $y = \sqrt[5]{\frac{(x-3)^4 (x^2+1)}{(2x+5)^3}}$

$\ln y = \ln \left(\frac{(x-3)^4 (x^2+1)}{(2x+5)^3} \right)^{1/5}$

$\ln y = \frac{1}{5} \ln \frac{(x-3)^4 (x^2+1)}{(2x+5)^3}$

$\ln y = \frac{1}{5} \left[\ln (x-3)^4 (x^2+1) - \ln (2x+5)^3 \right]$

$\ln y = \frac{1}{5} \left[\ln (x-3)^4 + \ln (x^2+1) - \ln (2x+5)^3 \right]$

$\ln y = \frac{1}{5} \left[4 \ln (x-3) + \ln (x^2+1) - 3 \ln (2x+5) \right]$

$y \cdot \frac{1}{y} \frac{dy}{dx} = \frac{1}{5} \left[\frac{4}{x-3} + \frac{2x}{x^2+1} - \frac{6}{2x+5} \right] \cdot \sqrt[5]{\text{etc.}}$

3.9

(45) $y = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$

$\ln y = \ln \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$

$\ln y = \frac{1}{5} \ln \frac{(x-3)^4(x^2+1)}{(2x+5)^3}$

$\ln y = \frac{1}{5}\left[\ln (x-3)^4(x^2+1) - \ln (2x+5)^3\right]$

$\ln y = \frac{1}{5}\left[\ln(x-3)^4 + \ln(x^2+1) - \ln(2x+5)^3\right]$

$\ln y = \frac{1}{5}\left[4\ln(x-3) + \ln(x^2+1) - 3\ln(2x+5)\right]$

$y \cdot \frac{1}{y}\frac{dy}{dx} = \frac{1}{5}\left[\frac{4}{x-3} + \frac{2x}{x^2+1} - \frac{6}{2x+5}\right] \sqrt[5]{\text{crap}}$

(47) $y = x^{\ln x}$

$\ln y = \ln x^{\ln x}$

$\ln y = \ln x \cdot \ln x$

$\ln y = (\ln x)^2$

$y \cdot \frac{1}{y}\frac{dy}{dx} = 2(\ln x)\left[\frac{1}{x}\right] \cdot x^{\ln x}$

$7x^3 \quad 7 \cdot 3x^2$

3.9

(25) $y = \ln 2 \cdot \log_2 x$

$\frac{dy}{dx} = \ln 2 \cdot \frac{1}{\ln 2 \, x} = \boxed{\frac{1}{x}}$

(16) $y = (\ln x)^2$

$y' = 2(\ln x)\left[\frac{1}{x}\right]$

$= \frac{2\ln x}{x}$ or $\frac{\ln(x^2)}{x}$

(30) $y = 2e^x - 1$

$\frac{dy}{dx} = 2e^x = \frac{1}{3} \cdot \frac{1}{2}$

$e^x = \frac{1}{6}$

$\ln e^x = \ln \frac{1}{6}$

$x \ln e = \ln \frac{1}{6}$

$x = \ln \frac{1}{6}$

$\perp \; y = -3x + 2$

need $m = \frac{1}{3}$

$y = 2e^{\ln \frac{1}{6}} - 1$

$2\left(\frac{1}{6}\right) - 1$

$\frac{1}{3} - 1 = -\frac{2}{3}$

$\boxed{\left(\ln \frac{1}{6}, -\frac{2}{3}\right)}$

(29) $y = 3^x + 1$

$y' = \frac{3^x \ln 3}{\ln 3} = \frac{5}{\ln 3}$

$\ln 3^x = \ln\left(\frac{5}{\ln 3}\right)$

$\frac{x \ln 3}{\ln 3} = \frac{\ln\left(\frac{5}{\ln 3}\right)}{\ln 3}$

$x = \frac{\ln\left(\frac{5}{\ln 3}\right)}{\ln 3}$

$\parallel \; y = 5x - 1$

need $m = 5$

$y = 3^{\frac{\ln\left(\frac{5}{\ln 3}\right)}{\ln 3}} + 1$

$\left(\frac{\ln\left(\frac{5}{\ln 3}\right)}{\ln 3}, 3^{\frac{\ln\left(\frac{5}{\ln 3}\right)}{\ln 3}} + 1\right)$

3.9

(43) $y = (\sin x)^x$

$\ln y = \ln (\sin x)^x$

$\ln y = \underset{f}{x} \underset{g}{\ln (\sin x)}$

$\frac{1}{y}\frac{dy}{dx} = \ln(\sin x) + \frac{1}{\sin x}[\cos x] \cdot x$

$\frac{1}{y}\frac{dy}{dx} = \ln(\sin x) + \frac{x\cos x}{\sin x}$

$y \cdot \frac{1}{y}\frac{dy}{dx} = \left[\ln(\sin x) + x\cot x\right] y$

$\frac{dy}{dx} = \left[\ln(\sin x) + x\cot x\right](\sin x)^x$

(47) $y = x^{\ln x}$ $\qquad (\ln x)^2$

$\ln y = \ln x^{(\ln x)}$

$\ln y = \underset{f}{\ln x} \cdot \underset{g}{\ln x}$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\ln x + \frac{1}{x}\ln x$

$\frac{1}{y}\frac{dy}{dx} = 2\,\frac{1}{x}\ln x$

$y\left[\frac{1}{y}\frac{dy}{dx}\right] = \left[\frac{2\ln x}{x}\right] y$

$$\boxed{\frac{dy}{dx} = \left[\frac{2\ln x}{x}\right] x^{\ln x}}$$