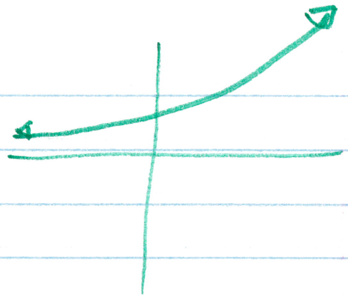
4.2

(19) $f(x) = e^{2x}$

$f'(x) = 2e^{2x} = 0$



(9) $f(x) = x + \frac{1}{x}$ $\quad [.5, 2]$

$(.5, 2.5)$

$(2, 2.5)$

$f'(x) = 1 + \frac{-1}{x^2} = 0$

$1 = \frac{1}{x^2}$

SECANT

$\boxed{y = 2.5}$ $\quad m = \frac{2.5 - 2.5}{5 - 2} = \underline{\underline{0}}$

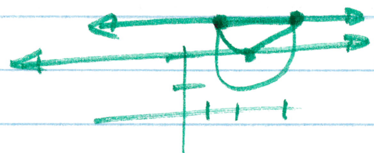
$x^2 = 1$

$x = \pm 1$

$f(1) = 1 + \frac{1}{1} = 2$

$x = 1$

TANGENT



$\boxed{y = 2}$

(5) $f(x) = \sin^{-1} x \quad [-1, 1]$

$(-1, \frac{-\pi}{2})$

$(1, \frac{\pi}{2})$

$f'(x) = \frac{1}{\sqrt{1 - x^2}} = \frac{\pi}{2}$ $f'(c)$

$\frac{\frac{\pi}{2} - -\frac{\pi}{2}}{1 - -1} = \frac{\frac{2\pi}{2}}{2} = \frac{\pi}{2}$

$\frac{f(b) - f(a)}{b - a}$

$\frac{2}{\pi} = \frac{\pi\sqrt{1 - x^2}}{\pi}$

$\left(\frac{2}{\pi}\right)^2 = \left(\sqrt{1 - x^2}\right)^2$

$\frac{4}{\pi^2} = 1 - x^2$

$\sqrt{x^2} = \pm\sqrt{1 - \frac{4}{\pi^2}}$

$x = \pm\sqrt{1 - \frac{4}{\pi^2}}$

$\boxed{c = \pm\sqrt{1 - \frac{4}{\pi^2}}}$

4.2 f g

(23) $f(x) = x\sqrt{4-x}$

$f'(x) = \sqrt{4-x} + \frac{1}{2}(4-x)^{-\frac{1}{2}}[-1]x$

$2\sqrt{4-x} = \frac{\sqrt{4-x}}{2\sqrt{4-x}} - \frac{x}{2\sqrt{4-x}}$

$\frac{2(4-x)}{2\sqrt{4-x}} - \frac{x}{2\sqrt{4-x}} = \frac{8-2x-x}{2\sqrt{4-x}} = \frac{8-3x}{2\sqrt{4-x}}$

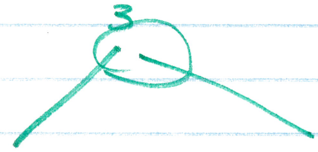
$8-3x=0$

$8=3x$

$\frac{8}{3}=x$

INCREASING	DECREASING
+	−
2	3

$\frac{8}{3}$ 4

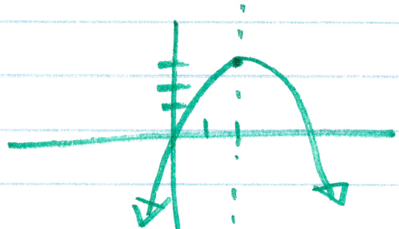


INCR	INCR
+	+

$\frac{8}{3}$

(40) $f(2)=3$, $f'(2)=0$

(a) $f'(x)>0$ for $x<2$, $f'(x)<0$ for $x>2$

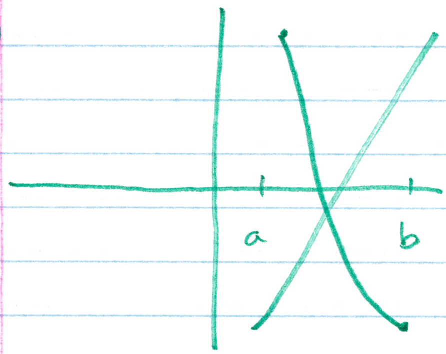


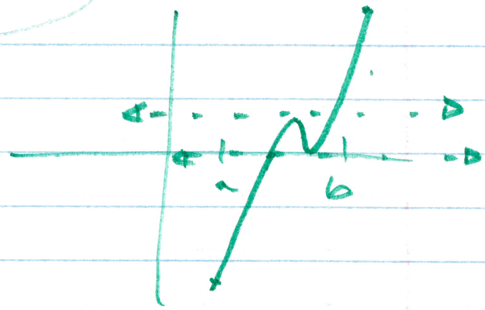
INCR DECR

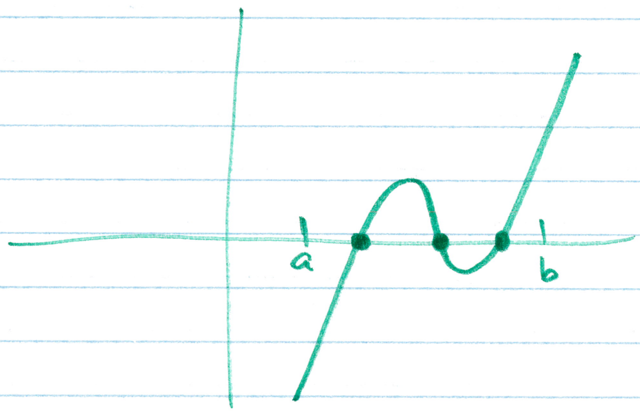
4.2

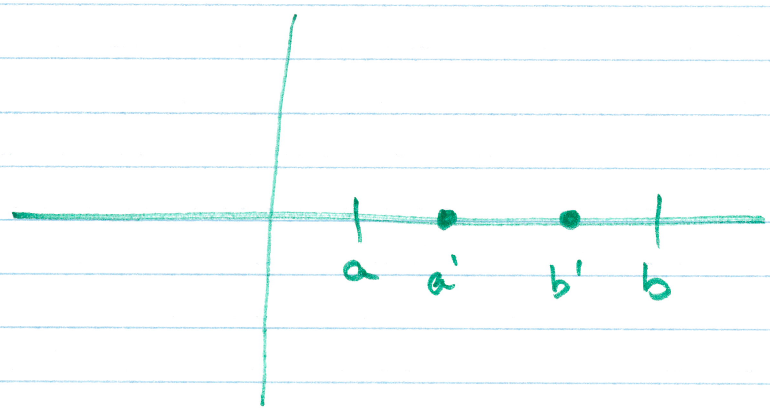
47

$f' \neq 0$







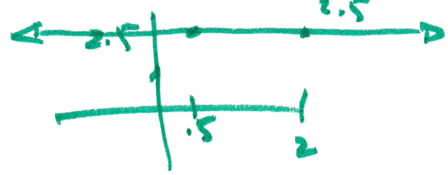


$$\frac{f(a')-f(b')}{a'-b'} = 0$$

$f'(c) =$

$f'(x) \neq 0$

SUPPOSE THERE ARE TWO ZEROES. THE AVERAGE RATE OF CHANGE BETWEEN THEM WOULD BE 0. BY THE MVT, SINCE $f(x)$ IS CONT. & DIFF. $f'(x) = 0$. BUT, $f'(x) \neq 0$ SO YOU CANNOT HAVE TWO (OR MORE) ZEROES.



x^{-1}

4.2

$(.5, 2.5) \quad (2, 2.5)$

(9) $f(x) = x + \frac{1}{x}$ $\quad [.5, 2]$

(a) $\frac{f(b) - f(a)}{b - a} = \frac{\left[2 + \frac{1}{2}\right] - \left[.5 + \frac{1}{.5}\right]}{2 - .5} = 0$ $\quad \boxed{y = 2.5}$

(b) $f'(x) = 1 + -\frac{1}{x^2} = 0$ $\quad f(1) = 1 + \frac{1}{1} = 2$

$1 = \frac{1}{x^2}$ $\quad \boxed{y = 2}$

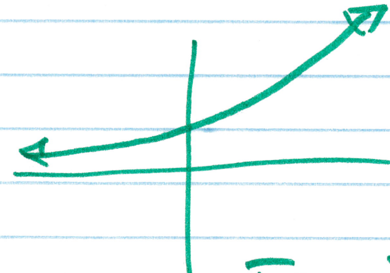
$x^2 = 1$

$x = \pm 1$

$\underline{x = 1}$

(19) $f(x) = e^{2x}$

$f'(x) = 2e^{2x} = 0$



(25) $h(x) = \frac{-x}{x^2 + 4}$ $\quad f \quad g$

$h'(x) = \frac{-(x^2+4) - (2x)(-x)}{(x^2+4)^2}$

$= \frac{-x^2 - 4 + 2x^2}{(x^2+4)^2}$

$= \boxed{\frac{x^2 - 4}{(x^2+4)^2}}$ $\quad (x^2+4)^2 \neq 0$

$x^2 - 4 = 0$ $\quad$ X

$x^2 = 4$

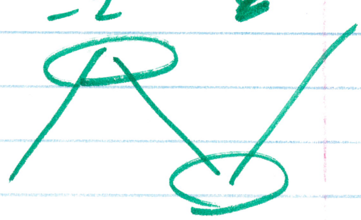
$x = \pm 2$

First Derivative Test

INCR	DECR	INCR
+	−	+
−3	0	3

−2 $\quad$ 2

−10 $\quad$ 10



4.2

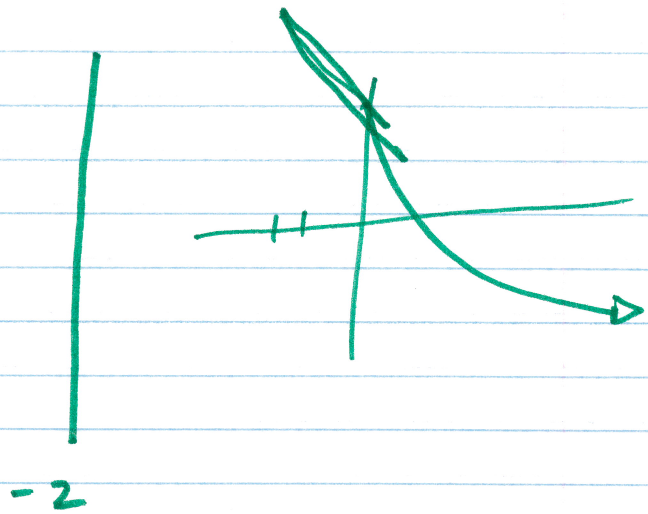
(21) $y = 4 - \sqrt{x+2}$

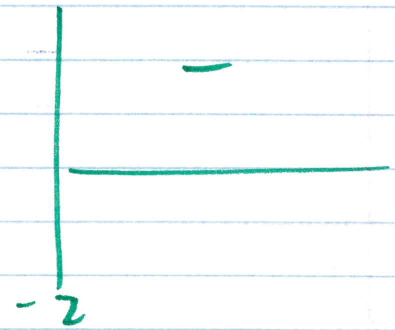
$y = 4 - (x+2)^{1/2}$

$y' = -\frac{1}{2}(x+2)^{-1/2}$

$= \dfrac{-1}{2(x+2)^{1/2}}$

$= \dfrac{-1}{2\sqrt{x+2}}$

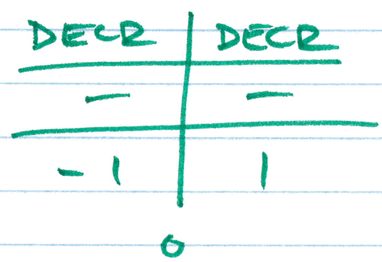


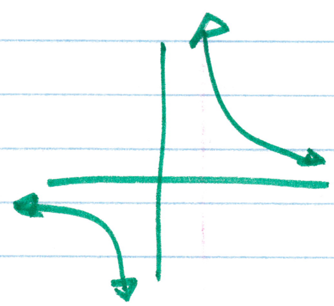


(17) $h(x) = \frac{2}{x}$

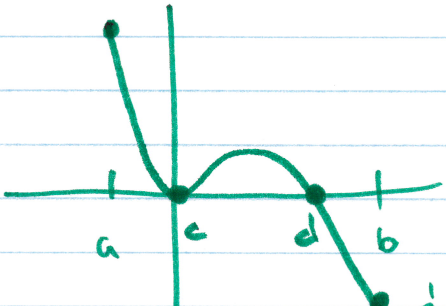
$h'(x) = -\frac{2}{x^2} = 0$

$x \neq 0$





(47)



$f' \neq 0$ Suppose you have two zeroes. Then the average ~~value~~ rate of change between them will be 0. By the MVT, since f is differentiable & continuous, we know $f'(x) = 0$ somewhere between the two zeroes. But $f' \neq 0$ is given. Therefore, there cannot be two zeroes of f on [a,b]. Q.E.D.