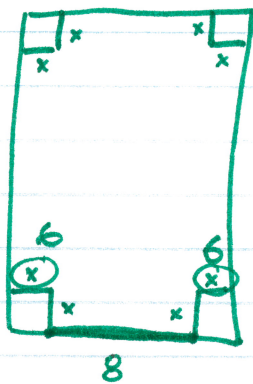
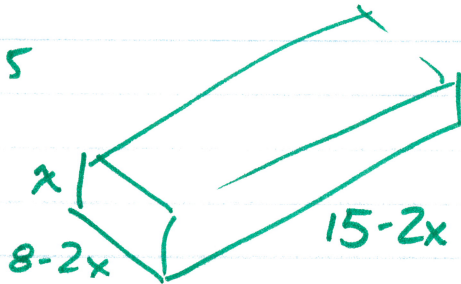


4.4

⑦



7.5



primary: $V = x(8-2x)(15-2x)$
 $= (8x-2x^2)(15-2x)$
 $= 120x - 16x^2 - 30x^2 + 4x^3$
 $= 120x - 46x^2 + 4x^3$

$V' = 120 - 92x + 12x^2 = 0$ $ax^2 + bx + c = 0$

$x = 6, \frac{2}{3}$

$12x^2 - 92x + 120 = 0$
 $4(3x^2 - 23x + 30) = 0$

$3x^2 - 23x + 30 = 0$

$(x - \frac{5}{3})(x - 6) = 0$

$(3x - 5)(x - 6) = 0$

$\frac{5}{3}, 6$

90	-23	
ac		b
-20	-3	
-5	-18	-23

FDT

	INCR	DECR	INCR
	+	-	+
	1	2	7
0	$\frac{5}{3}$	6	?

$$\frac{x^3}{x} - \frac{10x^2}{x} - \frac{30x}{x}$$

4.4

(25) $\frac{c(x)}{x} = \frac{x^3 - 10x^2 - 30x}{x} = x^2 - 10x - 30$

y

FDT	
DECR	INCR
-	+

$$\left(\frac{c(x)}{x}\right)' = 2x - 10 = 0$$

$x = 5$ THOUSAND

5

y'

(26) $\frac{c(x)}{x} = \frac{xe^x - 2x^2}{x} = e^x - 2x$

$$\left(\frac{c(x)}{x}\right)' = e^x - 2 = 0$$

$$\ln e^x = \ln 2$$

$$x \ln e = \ln 2$$

$$x = \ln 2$$

DECR	INCR
-	+
0	1

$\ln 2$

$\approx .693$ THOUSAND

4.4 primary

① (a) $x^2 + y^2 = S$

$(20-y)^2 + y^2 = S$

$2(20-y)' [-1] + 2y = S'$

$-40 + 2y + 2y =$

$-40 + 4y = 0$

$4y = 40$

$f(0) = 20^2 \quad 400$

$f(10) = 10^2 + 10^2 \quad 200 \quad y = 10$

$f(20) = 20^2 \quad 400$

Secondary

$x + y = 20$

$x = 20 - y$

LARGEST: 0, 20

SMALLEST: 10, 10

FDT

DECR	INCR
-	+

10



(b) $x + \sqrt{y} = S$

$\sqrt{x} + y = S$

$(20-y) + y^{1/2} = S$

$f(0) = 20$

$x \neq 0 \quad 0 - - 1 + \frac{1}{2} y^{-1/2} = S'$

$f(\frac{1}{4}) = 19\frac{3}{4} + \frac{1}{2} = 20\frac{1}{4}$

$1 = \frac{1}{2} y^{-1/2}$

$f(20) = \sqrt{20}$

$(y^{1/2})^2 = (\frac{1}{2})^2$

$y = \frac{1}{4}$

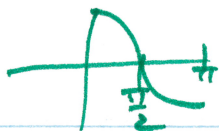
y x

→ $\frac{1}{4}, 19\frac{3}{4}$ LARGEST

→ 0, 20 SMALLEST
y x

constants

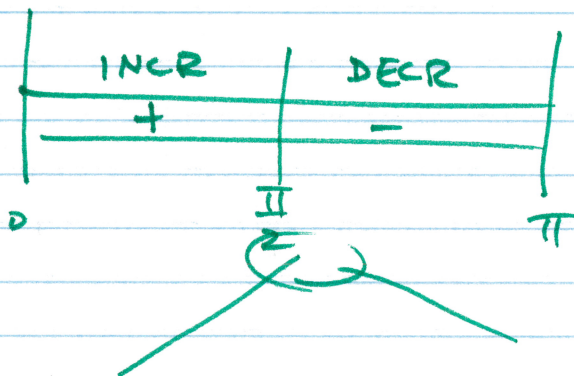
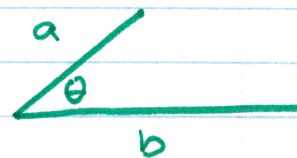
4.4 ↓↓



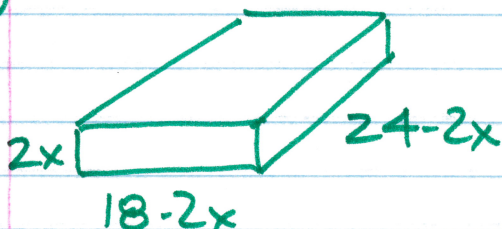
⑮ $A = \frac{1}{2} ab \sin \theta$

$$A' = \frac{1}{2} abc \cos \theta = 0$$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, \dots$



⑲



(a) $V = 2x(18-2x)(24-2x)$

(b) $(0, 9)$

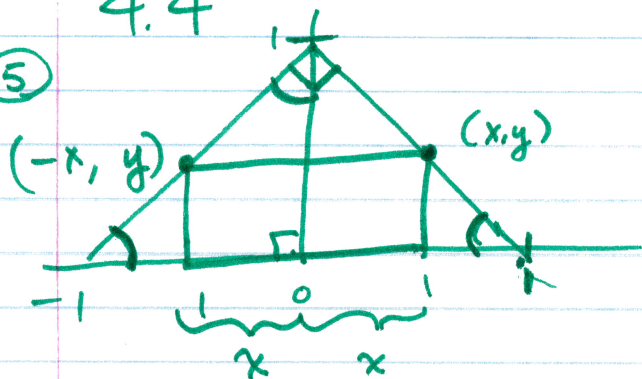
$$\begin{aligned} V(x) &= (36x - 4x^2)(24 - 2x) \\ &= 864x^2 - 72x^3 - 96x^2 + 8x^3 \\ &= 864x - 168x^2 + 8x^3 \end{aligned}$$

(c) $V'(x) = 864 - 336x + 24x^2 = 0$

(e) $864x - 168x^2 + 8x^3 = 1120$

4.4

(5)



$$y = mx + b$$

$$(a) y = -x + 1$$

$$(b) A = 2xy = 2x(-x + 1)$$

$$(c) A = -2x^2 + 2x$$

INCR	DECR
+	-

0 1/2 1

$$A' = -4x + 2 = 0$$

$$2 = 4x$$

$$\frac{1}{2} = x$$

profit = revenue - cost

$$(27) p(x) = \begin{cases} 200x - 32x - 6000 & 0 \leq x \leq 50 \\ [200 - 2(x - 50)]x - 32x - 6000 & 51 \leq x \leq 80 \end{cases}$$

$$p(x) = \begin{cases} 168x - 6000 & 0 \leq x \leq 50 \\ -2x^2 + 268x - 6000 & 51 \leq x \leq 80 \end{cases}$$

$$p'(x) = \begin{cases} 168 & 0 \leq x \leq 50 \\ -4x + 268 & 51 \leq x \leq 80 \end{cases}$$

$$(200 - 2x + 100)x$$

$$200x - 2x^2 + 100x - 32x$$

-6000

$$p(0) = ~~5400~~ - 6000$$

$$-4x + 268 = 0$$

$$268 = 4x$$

$$p(50) = 2400$$

$$p(51) = 2466$$

$$67 = x$$

$$p(67) = 2978$$

$$p(80) = 2640$$

4.4

31

(a)

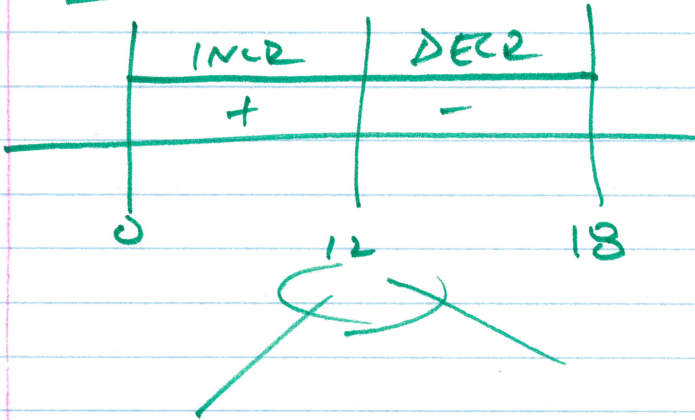


secondary $2x + 2y = 36$

$$\frac{x}{2\pi} = \frac{2\pi r}{2\pi}$$

$$2y = 36 - 2x$$

$$y = 18 - x$$



$$V = \pi r^2 h$$

$$V = \pi \left(\frac{x}{2\pi}\right)^2 y$$

$$V = \pi \frac{x^2}{4\pi^2} (18 - x)$$

$$V = \frac{1}{4\pi} (18x^2 - x^3)$$

$$V' = \frac{1}{4\pi} (36x - 3x^2) = 0$$

$$3x(12 - x) = 0$$

$$x = 0, 12$$

$$(12, 6)$$

AB
BC

4.4

$$\textcircled{24} \quad r(x) = \frac{x^2}{x^2+1}$$

$$c(x) = \frac{1}{3}(x-1)^3 - \frac{1}{3}$$

$$p = r - c$$

$$p(x) = \frac{x^2}{x^2+1} - \frac{1}{3}(x-1)^3 + \frac{1}{3}$$

$$p'(x) = \frac{2x(x^2+1) - 2x(x^2)}{(x^2+1)^2} - (x-1)^2$$

$$= \frac{2x^3 + 2x - 2x^3}{(x^2+1)^2} - (x-1)^2$$

$$= \frac{2x}{(x^2+1)^2} - (x-1)^2$$

4.4

⑤ (a) (1,0), (0,1)

$$m = \frac{0-1}{1-0} = -1$$

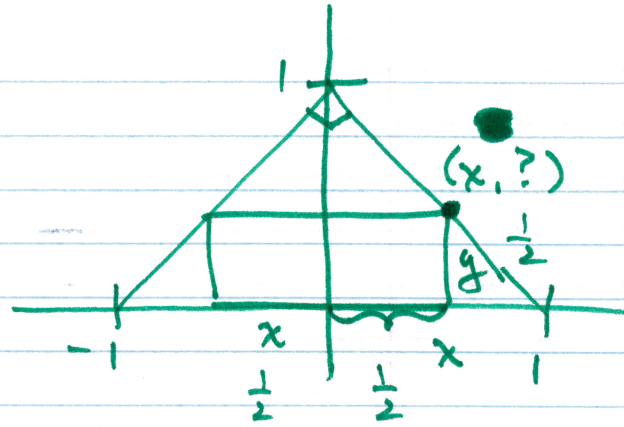
SECONDARY
 $y = -x + 1$

(b) $A = 2xy$
 $A = 2x(-x+1)$
 $A = -2x^2 + 2x$

(c) $A' = -4x + 2 = 0$
 $2 = 4x$
 $\frac{1}{2} = x$

$1 \times \frac{1}{2}$

AREA = $\frac{1}{2}$



FDT

INCR	DECR
+	-
$\frac{1}{4}$	$\frac{1}{2}$
$\frac{1}{2}$	

4.4

① $x = \text{a number}$
 $y = \text{another number}$

$x + y = 20$

$y = 20 - x$

(a) $x^2 + y^2 = s$

$x^2 + (20-x)^2 = s$

$2x + 2(20-x)[-1] = s'$

$2x - 2(20-x) = s'$

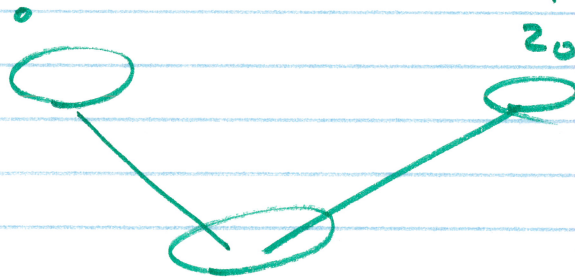
$2x - 40 + 2x = s'$

$4x - 40 = 0$

$x = 10$

FDT

DECR	INCR
-	+
9	11



SMALL : 10, 10

LARGE : 0, 20

(b) $z = x + \sqrt{y}$

$z = 20 - y + \sqrt{y}$

$z' = -1 + \frac{1}{2\sqrt{y}} = 0$

$\frac{1}{2\sqrt{y}} = 1$

$\frac{1}{\sqrt{y}} = 2$

$(\frac{1}{\sqrt{y}})^2 = (\frac{1}{2})^2$

$y = \frac{1}{4}$

$\frac{1}{\sqrt{y}} = 2$
 $\frac{1}{\sqrt{y}} = \frac{1}{2}$

$x + y = 20$

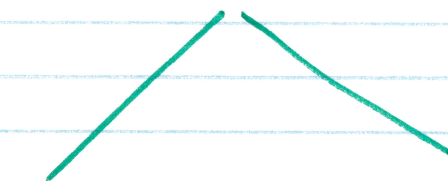
$y \neq 0$

$x = 20 - y$

$-1 + \frac{1}{2\sqrt{y}} = 0$

FDT

INCR	DECR
+	-
$\frac{1}{4}$	1

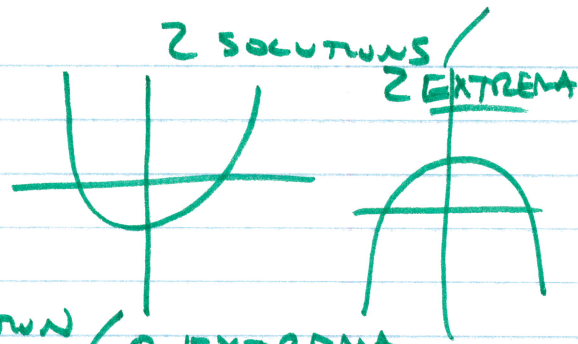


SMALL : 0, 20

LARGEST : $\frac{1}{4}, 19\frac{3}{4}$

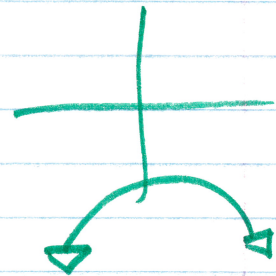
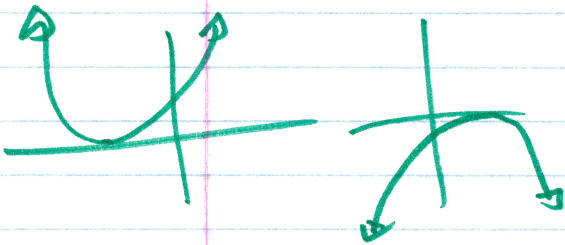
4.4

(29) $f(x) = ax^3 + bx^2 + cx + d$
 $f'(x) = 3ax^2 + 2bx + c$



1 SOLUTION / 0 EXTREMA

0 SOLUTION / 0 EXTREMA



$f'(x) = x^2 - 3$
 $f(x) = \frac{1}{3}x^3 - 3x$ 2 SOLUTIONS

$f'(x) = x^2 + 3$
 $f(x) = \frac{1}{3}x^3 + 3x$ 0 SOLUTIONS