6.2

29) $\frac{1}{4}\int \tan(4x+2)\, \overset{du}{dx\ 4}$ $\quad u = 4x+2$

$du = 4\,dx$

$\frac{1}{4}\int \tan u\, du = -\frac{1}{4}\int \frac{\sin u}{\cos u}\, du\,[-1]$ dv

$v = \cos u$

$dv = -\sin u\, du$

$-\frac{1}{4}\int \frac{1}{v}\, dv$

$-\frac{1}{4}\ln|v| + c$

$-\frac{1}{4}\ln|\cos u| + c$

$-\frac{1}{4}\ln|\cos(4x+2)| + c$

~~21) $\int \frac{dx}{x^2+9}$, $u = \frac{x}{3}$~~

$-\frac{1}{4}\ln|\cos(4x+2)| + c$ $\quad f(x) = x^2$

$-\frac{1}{4}\,\frac{1}{\cos(4x+2)}\,\underbrace{-\sin(4x+2)\,[4]}$ $\quad f(5) = \cancel{78}$

21) $\int \frac{dx}{x^2+9}$, $u = \frac{x}{3}$

$3u = x$

$3\,du = dx$

$\frac{1}{3}\tan^{-1} u + c = \boxed{\frac{1}{3}\tan^{-1}\frac{x}{3} + c}$

$\int \frac{3\,du}{(3u)^2+9} = \int \frac{3}{9u^2+9}\,\frac{du}{1} = \int \frac{3}{9(u^2+1)}\,du = \frac{1}{3}\int \frac{1}{u^2+1}\,du$

6.2

(43) $\int \frac{dx}{\cot 3x} = \frac{1}{3}\int \tan 3x \, \overbrace{dx[3]}^{du} = \frac{1}{3}\int \tan u \, du$

$u = 3x$

$du = 3dx$

$\frac{1}{3}\left[-\ln|\cos u| + c\right]$

$-\frac{1}{3}\ln|\cos 3x| + c$

$\frac{1}{3}\int \tan u \, du$

$-\frac{1}{3}\int \overbrace{\frac{\sin u}{\cos u} du[-1]}^{dv}$

$-\frac{1}{3}\int \frac{1}{v} dv$

$v = \cos u$

$dv = -\sin u \, du$

(45) $\int \sec x \, dx \left(\frac{\sec x + \tan x}{\sec x + \tan x}\right)$

$u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x \, dx$

$\int \overbrace{\frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx}^{du}$

$\int \frac{1}{u} du$

$\ln|u| + c$

$\boxed{\ln|\sec x + \tan x| + C}$

6.2

(47) $\int \sin^3 2x \quad , \quad \sin^2 2x = 1 - \cos^2 2x$

$$\int \underbrace{\sin^2 2x} \sin 2x \, dx$$

$$\int (1 - \cos^2 2x) \sin 2x \, dx$$

du [2] $\qquad$ $-[2]$ du

$$\frac{1}{2} \int \sin 2x \, dx + \frac{1}{2} \int \cos^2 2x \sin 2x \, dx$$

$u = 2x \qquad\qquad u = \cos 2x$

$du = 2\,dx \qquad\qquad du = -\sin 2x \,[2]\, dx$

$$\frac{1}{2} \int \sin u \, du + \frac{1}{2} \int u^2 \, du$$

$$-\frac{1}{2} \cos u + \frac{1}{2} \cdot \frac{1}{3} u^3 + C$$

$$-\frac{1}{2} \cos 2x + \frac{1}{6} \cos^3 2x + C$$

(53) $\int_0^3 (y+1)^{1/2} \, dy$

$\boxed{u = y + 1}$

$du = dy$

$$\int_1^4 u^{1/2} \, dy$$

$$\Big|_1^4 \frac{2}{3} u^{3/2}$$

$$\frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$$

$$\frac{16}{3} - \frac{2}{3} = \boxed{\frac{14}{3}}$$

6.2

53 $\int_0^3 \sqrt{y+1}\, dy$

$u = y+1$

$du = dy$

$\int_1^4 u^{1/2} du$

$\int_1^4 u^{1/2} du$

$\Big|_1^4 \frac{2}{3} u^{3/2}$

$\left[\frac{2}{3}(4)^{3/2}\right] - \left[\frac{2}{3}(1)^{3/2}\right]$

$\frac{16}{3} - \frac{2}{3} = \boxed{\frac{14}{3}}$

$\frac{1}{2} \ln \frac{13}{3}$

$\ln \left(\frac{13}{3}\right)^{\frac{1}{2}}$

$\ln \sqrt{\frac{13}{3}}$

47 $\int \sin^3 2x\, dx$

$\int \sin^2 2x \sin 2x\, dx$

$\int (1-\cos^2 2x) \sin 2x\, dx$

$\int (\sin 2x - \cos^2 2x \sin 2x)\, dx$

$\frac{1}{2}\int \sin 2x\, \overset{2}{dx} - \frac{1}{2}\int \cos^2 2x\, \underbrace{\sin 2x\, dx\, [-2]}_{du}$

$u = 2x$ $\quad$ $u = \cos 2x$

$du = 2dx$ $\quad$ $du = -\sin 2x\, [2]\, dx$

$\frac{1}{2}\int \sin u\, du + \frac{1}{2}\int u^2\, du$

$-\frac{1}{2}\cos u + \frac{1}{2}\cdot\frac{1}{3}u^3 + C$

$\boxed{-\frac{1}{2}\cos 2x + \frac{1}{6}\cos^3 2x + C}$

6.2

(21) $\int \frac{dx}{x^2+9}$; $u = \frac{x}{3}$

$3u = x$

$3du = dx$

$\int \frac{\cancel{dx}\, 3du}{(3u)^2+9}$

$\int \frac{\cancel{dx}\, 3du}{9u^2+9}$

$\int \frac{\cancel{dx}\, 3du}{9(u^2+1)}$

$\frac{3}{9} \int \frac{1}{u^2+1} \cancel{dx}\, du$

$\frac{1}{3} \arctan u + C$

$\boxed{\frac{1}{3} \arctan \frac{x}{3} + C}$

(6) $\int (2e^x + \sec x \tan x - \sqrt{x})\, dx$ $\quad x^{1/2}$

$\boxed{2e^x + \sec x - \frac{2}{3} x^{3/2} + C}$

(80) $\int 2 \sec^2 x \tan x\, dx$

(a) $u = \tan x$

$du = \sec^2 x\, dx$

$\int 2u\, du$

$u^2 + C$

$\boxed{\tan^2 x + C}$

$\int 2 \sec^2 x \tan x\, dx$

(b) $u = \sec x$

$du = \sec x \tan x\, dx$

$\int 2u\, du$

$u^2 + C$

$\boxed{\sec^2 x + C}$

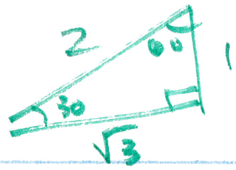
$- \tan^2 x + C$

$\sec^2 = \tan^2 + 1$

$\sec^2 x - \tan^2 x$

1

$\cancel{60} \cdot \frac{\pi}{\cancel{180}_{3}}$



$u = \tan^{-1}\sqrt{3}$

$u = \tan^{-1} 0$

6.2

(84) $\displaystyle\int_0^{\sqrt{3}} \frac{dx}{\sqrt{1+x^2}}$

$u = \tan^{-1} x$

$\tan u = \tan(\tan^{-1} x)$

$\cancel{\tan u = x}$ $\quad \tan u = x$

$\cancel{\tan 0 =}$ $\quad \sec^2 u = dx$

$\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sec^2 u \, du}{\sqrt{1+(\tan u)^2}}$

$\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sec^2 u \, du}{\sqrt{\sec^2 u}}$ $\qquad \sqrt{1+\tan^2 u}$

$\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sec^2 u \, du}{\sec u}$

$\displaystyle\int_0^{\frac{\pi}{3}} \sec u \, du$

(b) $\displaystyle\int_0^{\frac{\pi}{3}} \sec u \left(\frac{\sec u + \tan u}{\sec u + \tan u}\right) du$

$\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sec^2 u + \sec u \tan u}{\sec u + \tan u} \, du$

$v = \sec u + \tan u$

$dv = (\sec u \tan u + \sec^2 u) \, du$

$\displaystyle\int \frac{1}{v} \, dv$

$\ln |v|$

$\Big|_0^{\frac{\pi}{3}} \ln |\sec u + \tan u|$

$\ln |\sec \tfrac{\pi}{3} + \tan \tfrac{\pi}{3}| - \ln |\sec 0 + \tan 0|$

$\boxed{\ln |2+\sqrt{3}|} - \ln |1+0|$

6.2

33 $\int \frac{\ln^6 x}{x}\, dx$

$\int \frac{(\ln x)^6}{x}\, dx$

$\int (\ln x)^6 \left(\frac{1}{x}\, dx\right)$ du

$u = \ln x$

$du = \frac{1}{x}\, dx$

$\int u^6\, du$

$\frac{1}{7} u^7 + C$

$\boxed{\frac{1}{7} \ln^7 x + C}$

35 $\frac{3}{4}\int s^{1/3} \cos(s^{4/3} - 8)\, ds \left[\frac{4}{3}\right]$ du

$u = s^{4/3} - 8$

$du = \frac{4}{3} s^{1/3}\, ds$

$\frac{3}{4} \int \cos u\, du$

$\frac{3}{4} \sin u + C$

$\boxed{\frac{3}{4} \sin(s^{4/3} - 8) + C}$

57 $\frac{2}{3}\int_0^1 \frac{10\sqrt{\theta}}{(1+\theta^{3/2})^2}\, d\theta \left[\frac{3}{2}\right]$

$u = 1 + \theta^{3/2}$

$du = \frac{3}{2} \theta^{1/2}\, d\theta$

$\frac{2}{3} \int_1^2 \frac{10}{u^2}\, du$

$\frac{2}{3} \int_1^2 10u^{-2}\, du$

$\frac{2}{3} \Big|_1^2 -10\, u^{-1}$

$\Big|_1^2 -\frac{20}{3} u^{-1}$

$\frac{-20}{3}(2)^{-1} - \frac{-20}{3}(1)^{-1}$

$\frac{-20}{6} + \frac{20}{3} \qquad -\frac{20}{6} + \frac{40}{6} = \frac{20}{6} = \boxed{\frac{10}{3}}$

6.2

(73) $\int \tan x \, dx$

$-\int \frac{\sin x}{\cos x} \, dx \, [-]$ du

$u = \cos x$

$du = -\sin x \, dx$

$-\int \frac{1}{u} \, du$

$-\ln|u| + C$

$-\ln|\cos x| + C$

$\ln|\sec x| + C$

6.2

21 $\int \frac{dx}{x^2+9}$, $\boxed{u = \frac{x}{3}}$

$3u = x$

$3\,du = dx$

$\frac{dx}{9u^2+9}$

$\int \frac{3\,du}{9u^2+9}$

$\int \frac{3\,du}{9(u^2+1)}$

$\frac{1}{3}\int \frac{1}{u^2+1}\,du$

$\frac{1}{3}\arctan u + C$

$\boxed{\frac{1}{3}\arctan \frac{x}{3} + C}$

$\tan^{-1}$

$\frac{\int f(u)\,du}{\int f(u)\,dx}$

$\int k\,f(x)\,dx =$

$k\int f(x)\,dx$

II $\int \frac{dx}{x^2+9}$

$a^2 = x^2$

$u = x$

$a^2 = 9$

$a = 3$

$\frac{1}{a}\tan^{-1}\frac{u}{a} + C$

$\boxed{\frac{1}{3}\tan^{-1}\frac{x}{3} + C}$

$\int \frac{du}{u^2+a^2}$

$\int \frac{dx}{4x^2+25}$ $u = \frac{2x}{5}$ $u^2 = 4x^2$ $a^2 = 25$

$u = 2x$ $a = 5$

$\frac{5}{2}u = x$ $du = 2$

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57 $\frac{2}{3}\int_0^1 \frac{10\sqrt{\theta}}{(1+\theta^{3/2})^2}\,d\theta$ $[\frac{3}{2}]$

$u = 1+\theta^{3/2}$

$du = \frac{3}{2}\sqrt{\theta}\,d\theta$

$\frac{20}{3}\int_1^2 \frac{1}{u^2}\,du$ $\int u^{-2}$

$-u^{-1}$

$\frac{20}{3}\Big|_1^2 -\frac{1}{u}$

$\frac{20}{3}\left[-\frac{1}{2} - -\frac{1}{1}\right]$

$\frac{20}{3}\cdot\frac{1}{2}$

$\boxed{\frac{10}{3}}$