6.3

53) $\int \sin^{-1}x \, dx$

$u = \sin^{-1}x \qquad v = x$

$du = \frac{1}{\sqrt{1-x^2}}dx \quad dv = dx$

$x\sin^{-1}x \; - \frac{-1}{2}\int \frac{1}{\sqrt{1-x^2}} \overbrace{x\,dx[-2]}^{du}$

$u = 1-x^2$

$du = -2x\,dx$

$x\sin^{-1}x + \frac{1}{2}\int u^{-1/2}\,du$

$x\sin^{-1}x + \cancel{\frac{1}{2}} \cdot \cancel{2}u^{1/2} + C$

$\boxed{x\sin^{-1}x + \sqrt{1-x^2} + C}$

55) $\int \cos^{-1}x \, dx$

$u = \cos^{-1}x \qquad v = x$

$du = \frac{-1}{\sqrt{1-x^2}}dx \quad dv = dx$

$x\cos^{-1}x - \frac{1}{2}\int \overbrace{\frac{-1}{\sqrt{1-x^2}}x\,dx[2]}^{du}$

$u = 1-x^2$

$du = -2x\,dx$

$x\cos^{-1}x - \frac{1}{2}\int u^{-1/2}\,du$

$x\cos^{-1}x - \cancel{\frac{1}{2}} \cdot \cancel{2}u^{1/2} + C \; = \boxed{x\cos^{-1}x - \sqrt{1-x^2} + C}$

6.3

(19) $\int e^x \cos 2x \, dx$

$u = \cos 2x \qquad v = e^x$

$du = -2\sin 2x \, dx \qquad dv = e^x dx$

$\int e^x \cos 2x \, dx = e^x \cos 2x + \int +2e^x \sin 2x \, dx$

$u = \sin 2x \qquad v = e^x$

$du = 2\cos 2x \, dx \qquad dv = e^x dx$

$\int e^x \cos 2x \, dx = e^x \cos 2x + 2\left[e^x \sin 2x - \int 2e^x \cos 2x \, dx\right]$

$\int e^x \cos 2x \, dx = e^x \cos 2x + 2e^x \sin 2x - \cancel{4\int e^x \cos 2x \, dx}$

$+4\int e^x \cos 2x \, dx \qquad \cancel{+4\int e^x \cos 2x \, dx}$

$\frac{\cancel{5}\int e^x \cos 2x \, dx}{\cancel{5}} = \frac{e^x \cos 2x + 2e^x \sin 2x}{5}$

$\int e^x \cos 2x \, dx = \boxed{\frac{1}{5}e^x \cos 2x + \frac{2}{5}e^x \sin 2x + C}$

(31) $\frac{dy}{d\theta} = \theta \sec^{-1}\theta$

$\int dy = \int \theta \sec^{-1}\theta \, d\theta$

$u = \sec^{-1}\theta \qquad v = \frac{1}{2}\theta^2$

$y = \qquad du = \frac{1}{\theta\sqrt{\theta^2-1}} d\theta \qquad dv = \theta \, d\theta$

du

$y = \frac{1}{2}\theta^2 \sec^{-1}\theta - \frac{1}{4}\int \frac{1}{\cancel{\theta}\sqrt{\theta^2-1}} \cdot \frac{1}{2}\cancel{\theta} \, d\theta \; [4]$

$u = \theta^2 - 1$

$du = 2\theta \, d\theta$

6.3

31 cont. $y = \frac{1}{2}\theta^2 \sec^{-1}\theta - \frac{1}{4}\int u^{-1/2}\, du$

$y = \frac{1}{2}\theta^2 \sec^{-1}\theta - \frac{1}{\cancel{4}_2} \cancel{2} u^{1/2} + C$

$$\boxed{y = \frac{1}{2}\theta^2 \sec^{-1}\theta - \frac{1}{2}\sqrt{\theta^2 - 1} + C}$$

33 (a) $\int_0^{\pi} x \sin x \, dx$

$u = x \qquad v = -\cos x$

$du = dx \qquad dv = \sin x \, dx$

$\Big|_0^{\pi} -x\cos x + \int_0^{\pi} +\cos x \, dx$

$\Big|_0^{\pi} -x\cos x + \sin x$

$[-\pi\cos\pi + \sin\pi] - [-0\cos 0 + \sin 0]$

$[\pi + 0] - [0 + 0] = \pi$

35 $\frac{1}{2\pi - 0}\int_0^{2\pi} 2e^{-t}\cos t \, dt$

$u = \cos t \qquad v = -e^{-t}$

$\cancel{\frac{2}{2\pi}}$ $\quad du = -\sin t \qquad dv = e^{-t}dt$

$\frac{1}{\pi}\left[-\cos t e^{-t} - \int e^{-t}\sin t \, dt\right]$

$u = \sin t \qquad v = -e^{-t}$

$du = \cos t \, dt \qquad dv = e^{-t}\, dt$

6.3

35 cont. $\frac{1}{\pi}\left[-\cos t\, e^{-t} - \left[-e^{-t}\sin t - \int -e^{-t}\cos t\, dt\right]\right]$

$\frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t\, dt = -\frac{1}{\pi}\cos t\, e^{-t} + \frac{1}{\pi}e^{-t}\sin t - \frac{1}{\pi}\int e^{-t}\cos t\, dt$

$+ \frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t\, dt \qquad + \frac{1}{\pi}\int e^{-t}\cos t\, dt$

$\frac{\pi}{2} \quad \frac{2}{\pi}\int_0^{2\pi} e^{-t}\cos t\, dt = \left[-\frac{1}{\pi}\cos t\, e^{-t} + \frac{1}{\pi}e^{-t}\sin t\right]\frac{\pi}{2}$

$= \left. -\frac{1}{2}\cos t\, e^{-t} + \frac{1}{2}e^{-t}\sin t \right|_0^{2\pi}$

$\left[-\frac{1}{2}\overset{1}{\cos 2\pi}\, e^{-2\pi} + \cancel{\frac{1}{2}e^{-2\pi}\sin 2\pi}\right] -$

$\left[-\frac{1}{2}\overset{1}{\cos 0}\, e^{-0} + \cancel{\frac{1}{2}e^{-0}\sin 0}\right]$

$\boxed{-\frac{1}{2}e^{-2\pi} + \frac{1}{2}}$

6.3

(4) $\int 2t\cos(3t)\,dt \quad = \quad uv - \int v\,du$

$u = 2t \qquad v = \frac{1}{3}\sin(3t) \qquad \frac{2}{3}t\sin(3t) - \int \frac{2}{3}\sin(3t)\,dt$

$du = 2\,dt \qquad dv = \cos(3t)\,dt \qquad \boxed{\frac{2}{3}t\sin(3t) + \frac{2}{9}\cos(3t) + C}$