6.4

⑦ $\frac{dy}{dx} = (\cos x)e^{y+\sin x}$

$\frac{dx}{e^y}\frac{dy}{dx} = (\cos x)e^{y}e^{\sin x}\frac{dx}{e^{y}}$

$\int e^{-y}\,dy = \int e^{\sin x}\cos x\,dx$

$-e^{-y} = \int e^{u}\,du$

$u = \sin x$

$du = \cos x\,dx$

$= e^{u}$

$-e^{-y} = e^{\sin x} + C$

$\ln e^{-y} = \ln(-e^{\sin x} + C)$

$\pm y = -\ln(-e^{\sin x} + C)$

$y = -\ln(-e^{\sin x} + 2)$

$0 = -\ln(-e^{\sin 0} + C)$

$-0 = +\ln(-1 + C)$

$e^{0} = -1 + C$

$1 = -1 + C$

$2 = C$

$x^2 \cdot x^3 = x^{2+3} = x^5$

$-\int e^{-y}(-1)\,dy = \int du$

$u = -y$

$du = -dy$

$-\int e^{u}\,du$

$-e^{u}$

$-e^{-y}$

$$\frac{dy}{dt} = ky$$

6.4

(21) $\frac{dy}{dt} = -.0077y$ $\quad k$

$$y = Ce^{kt}$$

$$y = Ce^{-.0077t}$$

$$\frac{y}{C} = \frac{Ce^{-.0077t}}{C}$$

$$\frac{1}{2} = e^{-.0077t}$$

$$\ln\frac{1}{2} = \ln e^{-.0077t}$$

$$\frac{\ln\frac{1}{2}}{-.0077} = \frac{-.0077t}{-.0077}$$

$a \log x = \log x^a$

6.4

(16) $y = Ce^{kt}$

$y = 2000e^{kt}$

$\frac{4000}{2000} = \frac{2000e^{k(15)}}{2000}$

$2 = e^{15k}$

$\ln 2 = \ln e^{15k}$

$\frac{\ln 2}{15} = \frac{15k}{15}$

$= k$

$y = 2000e^{\frac{\ln 2}{15}t}$

$y = 2000e^{\frac{\ln 2}{15} \cdot 30}$

$= 2000e^{(\ln 2)2} \rightarrow 2000e^{2\ln 2}$

$= 2000e^{\ln 2^2}$

$= 2000e^{\ln 4}$

$= 2000 \cdot 4 = \boxed{\$8000}$

(7) $\frac{dy}{dx} = (\cos x)e^{y+\sin x}$ $\quad y=0, x=0$

$\frac{dy}{dx} = (\cos x)e^{y}e^{\sin x}$

$\frac{dy}{e^y} = (\cos x)e^{\sin x}\,dx$

$-\int e^{-y}\,dy = \int (\cos x)e^{\sin x}\,dx$ $\quad du$

$u = -y$ $\qquad u = \sin x$

$du = -1\,dy$ $\qquad du = \cos x\,dx$

$-\int e^u\,du \qquad \int e^u\,du$

$-e^u \qquad e^u$

$-e^{-y} = e^{\sin x} + C \rightarrow -e^{-y} = e^{\sin x} - 2$

$-e^{-0} = e^{\sin 0} + C$

$-1 = 1 + C$

$-2 = C$

$\ln e^{-y} = \ln(-e^{\sin x} + 2)$

$-y = \ln(-e^{\sin x} + 2)$

$y = -\ln(-e^{\sin x} + 2)$

$y = \ln \frac{1}{-e^{\sin x} + 2}$

6.4

(32) $\ln|y-y_c| = kt + C$ $\quad\quad$ $\ln|y-y_c| = kt + \ln 60$

$\ln|60| = k(0) + C$ $\quad\quad$ $\ln 70 = k(-20) + \ln 60$

$\ln 60 = C$ $\quad\quad$ $-\ln 60 \quad\quad -\ln 60$

$\frac{\ln \frac{7}{6}}{-20} = \frac{-20k}{-20}$

$\ln|\widetilde{y-y_c}| = \frac{\ln \frac{7}{6}}{-20} t + \ln 60$

(a) $\ln|y-y_c| = \frac{\ln \frac{7}{6}}{-20}(15) + \ln 60$ $\longrightarrow$ $\ln|y-y_c| = \ln\left(\frac{7}{6}\right)^{-\frac{3}{4}} + \ln 60$

$\ln|y-y_c| = \ln\left[\left(\frac{7}{6}\right)^{-\frac{3}{4}} \cdot 60\right]$

$|y-y_c| = e^{\frac{\ln \frac{7}{6}}{-20}(15) + \ln 60}$ $\quad\quad$ $|y-y_c| = \left(\frac{7}{6}\right)^{-\frac{3}{4}} \cdot 60$

(b) $\ln|y-y_c| = \frac{\ln \frac{7}{6}}{-20}(120) + \ln 60$

$\ln|y-y_c| = -6\ln \frac{7}{6} + \ln 60$

$\ln|y-y_c| = \ln\left(\frac{7}{6}\right)^{-6} + \ln 60$

$\ln|y-y_c| = \ln\left[\left(\frac{7}{6}\right)^{-6} \cdot 60\right]$

$|y-y_c| = \left(\frac{7}{6}\right)^{-6} \cdot 60$

(c) $\ln|10| = \frac{\ln \frac{7}{6}}{-20} t + \ln 60$

$-\ln 60 \quad\quad -\ln 60$

$\ln \frac{1}{6} = \frac{\ln \frac{7}{6}}{-20} t$

$\frac{-20 \ln \frac{1}{6}}{\ln \frac{7}{6}} = t$

6.4

(30) $\ln|y-y_e| = kt+C$

$\ln|y-65| = kt+C$

$-(\ln|35-65| = k(10)+C)$

$(\ln|50-65| = k(20)+C)$

$\ln 15 - \ln 30 = 10k$

$\ln\frac{15}{30} = 10k$

$\frac{\ln\frac{1}{2}}{10} = k$

$\ln 30 = \frac{\ln\frac{1}{2}}{10}\cdot 10 + C$

$\ln 30 = \ln\frac{1}{2} + C$

$-\ln\frac{1}{2} \quad -\ln\frac{1}{2}$

$\ln\frac{30}{\frac{1}{2}} = C$

$\ln 60 = C$

$\ln|y-65| = \frac{\ln\frac{1}{2}}{10}t + \ln 60$

$\ln|y-65| = \frac{\ln\frac{1}{2}}{10}(0) + \ln 60$

$\ln|y-65| = \ln 60$

$|y-65| = 60$

$y-65 = 60$

$y = 125°F$

$y-65 = -60$

$y = 5°F$

6.4

(23)

1/2 HOUR	$\times 2$
2/2 HOUR	$\times 2^2$
1 1/2 HOUR	$\times 2^3$
2 HOUR	$\times 2^4$
⋮	
24 HOUR	$2^{48} \approx 2.8 \times 10^{14}$

$2,800,000,000,000$

(10) $\frac{dy}{dx} = \frac{4\sqrt{y}\ln x}{x}$ $y=1,\ x=e$

$\int \frac{1}{\sqrt{y}}\,dy = \int \frac{4\ln x}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int y^{-\frac{1}{2}}\,dy = \int 4u\,du$

$2\sqrt{y} = \int 4u\,du$

$2\sqrt{y} = 2u^2 + C$

$\frac{2\sqrt{y}}{2} = \frac{2\ln^2 x + C}{2}$

$\sqrt{y}^2 = (\ln^2 x + C)^2$

$y = (\ln^2 x + C)^2$ $\rightarrow$ $1 = (\ln e)^2$

$\sqrt{1} = \sqrt{((\ln e)^2 + C)^2}$

$\pm 1 = 1 + C$

$0, -2 = C$

$y = (\ln^2 x)^2$

$y = \ln^4 x$

$(x^2)^4 = x^8$

$x^2 \cdot x^4 = x^6$

6.4

(22) $y = Ce^{kt}$ ← $\frac{dy}{dt} = ky$

$\frac{y}{C} = \frac{1}{2} = e^{kt}$

$\frac{1}{2} = e^{k(65)}$

$\ln \frac{1}{2} = \ln e^{65k}$

$\frac{\ln \frac{1}{2}}{65} = \frac{65k}{65}$

6.4

(11) $\frac{dy}{dt} = ky \rightarrow y = Ce^{kt}$

$y = Ce^{1.5t}$ $\quad y(0) = 100$

$\boxed{y = 100e^{1.5t}}$

$100 = Ce^{1.5(0)}$

$100 = C$

(14) $y(0) = 60, \; y(10) = 30$

$y = 60e^{kt}$

$\frac{30}{60} = \frac{60e^{10k}}{60}$

$\frac{1}{2} = e^{10k}$

$\ln \frac{1}{2} = \ln e^{10k}$

$\frac{\ln \frac{1}{2}}{10} = \frac{10k}{10}$

$y = 60e^{\frac{\ln \frac{1}{2}}{10} t}$