8.4

(13) $\int_{-1}^{\infty} \frac{dx}{x^2+5x+6} = \lim_{b\to\infty} \int_{-1}^{b} \left[\frac{1}{x+2} - \frac{1}{x+3}\right] dx$

$\lim_{b\to\infty} \Big|_{-1}^{b} \ln|x+2| - \ln|x+3|$

$\lim_{b\to\infty} \left[\ln|b+2| - \ln|b+3|\right] - \left[\ln|-1+2| - \ln|-1+3|\right]$

$\lim_{b\to\infty} \left[\ln\left(\frac{b+2}{b+3}\right)\right] \quad 0 - [0 - \ln 2] = \boxed{\ln 2}$

$\qquad 1$

$\frac{1}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2} = \frac{1}{(x+3)(x+2)}$

$A(x+2) + B(x+3) = 1$

$x = -2 \quad B = 1$

$x = -3 \quad -A = 1$

$A = -1$

8.4

(15) $\int_1^{\infty} \frac{5x+6}{x^2+2x}\,dx = \lim_{b\to\infty} \int_1^b \left(\frac{3}{x} + \frac{2}{x+2}\right) dx$

$\lim_{b\to\infty} \Big|_1^b\ 3\ln|x| + 2\ln|x+2|$

$\lim_{b\to\infty} \left[3\ln b + 2\ln|b+2|\right] - \left[3\ln 1 + 2\ln 3\right]$

DIVERGES

$\lim_{x\to\infty} x^2 - x = \cancel{0}\ \infty$

$\lim_{x\to\infty} \frac{x^2 - \cancel{x}}{\cancel{3}x^2 - \cancel{x}} \qquad \frac{1}{3}$

$\frac{5x+6}{x^2+2x} = \frac{\quad}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$

$A(x+2) + Bx = 5x+6$

$x=-2 \qquad -2B = -4 \longrightarrow B = 2$

$x=0 \qquad 2A = \cancel{8}6 \longrightarrow A = 3$

8.4

(19) $\int_1^{\infty} x \ln(x)\, dx$

$$\lim_{b\to\infty} \Big[\tfrac{1}{2}x^2 \ln x - \tfrac{1}{4}x^2 \Big]_1^{b}$$

$$\lim_{b\to\infty} \Big[\underbrace{\tfrac{1}{2}b^2}\ln b - \underbrace{\tfrac{1}{4}b^2} \Big] - \Big[\overset{0}{\tfrac{1}{2}(1)^2 \ln 1} \overset{-\frac{1}{4}}{- \tfrac{1}{4}(1)^2} \Big]$$

$\boxed{\text{DIVERGES}}$

$$\lim_{b\to\infty} \frac{\tfrac{1}{2}\cancel{b^2}\ln b}{\tfrac{1}{4}\cancel{b^2}}$$

$u = \ln x \qquad v = \tfrac{1}{2}x^2$

$du = \tfrac{1}{x}dx \qquad dv = x\, dx$

$\tfrac{1}{2}x^2 \ln x - \int \tfrac{1}{2}x^2 \cdot \tfrac{1}{x}\, dx$

$\tfrac{1}{2}x^2 \ln x - \int \tfrac{1}{2}x\, dx$

$\tfrac{1}{2}x^2 \ln x - \tfrac{1}{4}x^2$

8.4

25 $\int_0^2 \frac{dx}{1-x^2} = \int_0^1 \frac{dx}{1-x^2} + \int_1^2 \frac{dx}{1-x^2}$

$\lim_{b\to 1^-} \int_0^b \left[\frac{1}{2(1-x)} + \frac{1}{2(1+x)}\right]dx + \lim_{a\to 1^+} \int_a^2 \left[\frac{1}{2(1-x)} + \frac{1}{2(1+x)}\right]dx$

$\lim_{b\to 1^-} \Big|_0^b \left[\boxed{\tfrac{1}{2}}\ln|(1-x)| + \tfrac{1}{2}\ln|1+x|\right] + \lim_{a\to 1^+} \Big|_a^2 \left[\boxed{\tfrac{1}{2}}\ln|1-x| + \tfrac{1}{2}\ln|1+x|\right]$

$-\infty \qquad \cancel{\tfrac{1}{2}\ln 2} \qquad 0$

$\lim_{b\to 1^-} \left[\tfrac{1}{2}\ln|1-b| + \cancel{\tfrac{1}{2}\ln|1+b|}\right] - \left[\cancel{\tfrac{1}{2}\ln|1|} + \tfrac{1}{2}\ln|1|\right]$

0

$+ \lim_{a\to 1^+} \left[\tfrac{1}{2}\cancel{\ln}|1-2| + \tfrac{1}{2}\ln|1+2|\right] - \left[\tfrac{1}{2}\ln|1-a| + \tfrac{1}{2}\ln|1+a|\right]$

$\to \tfrac{1}{2}\ln 3 \qquad -\infty \qquad \cancel{\tfrac{1}{2}\ln 2}$

?

$\lim_{b\to 1^-} \tfrac{1}{2}\ln\frac{1-b}{1+b} \qquad \tfrac{1}{2}\ln\frac{1-a}{1+a}$

$\frac{1}{1-x^2} = \frac{A}{(1-x)} + \frac{B}{(1+x)}$

$(1)^2-(x)^2 \qquad A(1+x)+B(1-x)=1$

$x=-1 \quad 2B=1 \quad B=\tfrac{1}{2} \qquad x=1 \quad 2A=1 \quad A=\tfrac{1}{2}$

8.4

25 TAKE 2

$$\lim_{b\to 1^-} \Big|_0^b \frac{1}{2}\ln\frac{1-x}{1+x} + \lim_{a\to 1^+} \Big|_a^2 \frac{1}{2}\ln\frac{1-x}{1+x}$$

$$\lim_{b\to 1^-} \frac{1}{2}\left[\ln\frac{1-b}{1+b} - \ln\frac{1-0}{1+0}\right] + \lim_{a\to 1^+} \frac{1}{2}\left[\ln\frac{1-2}{1+2} - \ln\frac{1-a}{1+a}\right]$$

$$-\infty \qquad 0 \qquad \ln\frac{1}{3} \qquad - -\infty$$

?

8.4

$$\int_0^1 \frac{x+1}{\sqrt{x^2+2x}}\,dx = \lim_{a\to 0^+} \frac{1}{2}\int_a^1 \frac{[2]\,x+1}{\sqrt{x^2+2x}}\,dx \quad du$$

$$\lim_{a\to 0^+} \cancel{\tfrac{1}{2}}\Big|_a^1 \cancel{2}\,u^{1/2}$$

$$\lim_{a\to 0^+} \Big|_a^1 \sqrt{x^2+2x}$$

$$\lim_{a\to 0^+} \left[\sqrt{1^2+2(1)} - \cancel{\sqrt{a^2+2a}}^{\,0}\right]$$

$$\boxed{\sqrt{3}}$$

$u = x^2+2x$

$du = (2x+2)\,dx \quad \int \frac{1}{\sqrt{u}} \quad \int u^{-1/2} \quad \underline{2}\,u^{1/2}$

$= \underline{2}(x+1)\,dx$

$\cancel{\ln \sqrt{u} = \frac{1}{\sqrt{u}}\left[\frac{1}{2}u^{-1/2}\right]}$