9.1

(23) $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n (x-3)^n$

$\sum_{n=0}^{\infty} \left(-\frac{1}{2}(x-3)\right)^n$ at $x=4$ $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n$

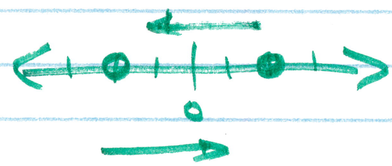
$\left| -\frac{1}{2}(x-3) \right| < 1$

$|r| < 1$

$2 \left| \frac{1}{2}(x-3) \right| < 1 \cdot 2$

$-1 < r < 1$

$|(x-3)| < 2$



$x-3 < 2$

$x < 5$

$x-3 > -2$

$x > 1$

$\boxed{(1,5)}$ INTERVAL OF CONVERGENCE

$|x| < 2$

-1

9.1

17 $\sum_{n=0}^{\infty} \sin^n\left(\frac{\pi}{4}+n\pi\right)$

$\sin^0\left(\frac{\pi}{4}+0\pi\right) + \sin\left(\frac{\pi}{4}+\pi\right) + \sin^2\left(\frac{\pi}{4}+2\pi\right) + \sin^3\left(\frac{\pi}{4}+3\pi\right)$

$1 \quad - \frac{\sqrt{2}}{2} \quad + \frac{1}{2} \quad - \frac{\sqrt{2}}{4}$

$-\frac{\sqrt{2}}{2} \quad -\frac{\sqrt{2}}{2} \quad -\frac{\sqrt{2}}{2}$

$\frac{2}{4}$

$\frac{1}{1-\frac{\sqrt{2}}{2}} \qquad \frac{1}{\frac{2+\sqrt{2}}{2}} \qquad \boxed{\frac{2}{2+\sqrt{2}}}$

$\frac{1}{1+\frac{1}{\sqrt{2}}} \qquad \frac{1}{\frac{\sqrt{2}+1}{\sqrt{2}}} \qquad \frac{\sqrt{2}}{1+\sqrt{2}} \cdot \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}-2}{1-2}$

$\boxed{-\sqrt{2}+2}$

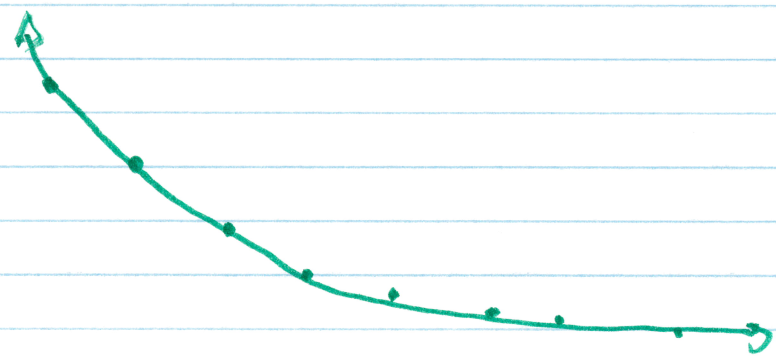
#21

31 $\sum_{n=0}^{\infty} 2^n x^n \qquad \int_0^x f(t)\,dt$

$\sum_{n=0}^{\infty} (2x)^n \qquad \int_0^x \frac{1}{1-2t}\,dt$

$f(x) = \frac{1}{1-2x} \qquad \int_0^x f(t)\,dt = \frac{1}{2}\ln|1-2x|$ as series: $\sum_{n=0}^{\infty} \frac{1}{2}\cdot\frac{1}{n+1}(2x)^{n+1}$

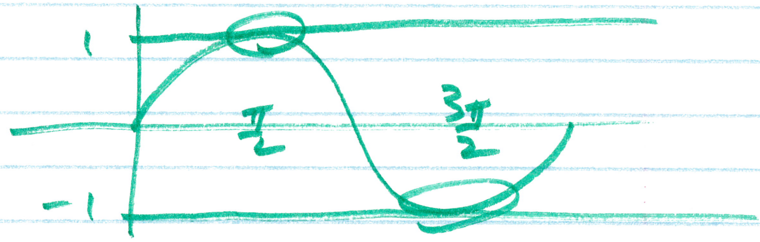
$\sum_{n=0}^{\infty} \frac{1}{2n+2}(2x)^{n+1}$



9.1

25) $\sum_{n=0}^{\infty} \sin^n x$

$\sum_{n=0}^{\infty} (\sin x)^n$



$|\sin x| < 1$

$f(x) = \frac{1}{1 - \sin x}$

$x \neq \frac{\pi}{2} + \pi n$

27) #21

$\sum_{n=0}^{\infty} (2x)^n$

$\sum_{n=0}^{\infty} 2n\,(2x)^{n-1}$

$f(x) = \frac{1}{1-2x} = (1-2x)^{-1}$ $f'(x) = \frac{+2}{(1-2x)^2}$

71) $\sum_{n=0}^{\infty} (x-1)^n$

$\cancel{\int_0^x (t-1)^n\,dt}$

$\int_0^x f(t)\,dt$

$\frac{1}{1-(x-1)}$

$-\int \frac{1}{2-x} \cdot -dx$

$-\ln|2-x|$

$u = 2-x$

$du = -dx$