9.2

(1) $f(x)=\sqrt{1+x^2}$; $x=0$

$g(x)=\sqrt{1+x} = (1+x)^{\frac{1}{2}}$ $\quad g(0)=1$

$g'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}}$ $\quad g'(0)=\frac{1}{2}$

$g''(x) = -\frac{1}{4}(1+x)^{-\frac{3}{2}}$ $\quad g''(0)=-\frac{1}{4}$

$g'''(x) = +\frac{3}{8}(1+x)^{-\frac{5}{2}}$ $\quad g'''(0)=\frac{3}{8}$

$g^4(x) = \frac{-15}{16}(1+x)^{-\frac{7}{2}}$ $\quad g^4(0)=\frac{-15}{16}$

$\frac{2k!}{k!}$

$\frac{(2k+1)!}{k!}$

$$1+\frac{1}{2}x-\frac{\frac{1}{4}x^2}{\underset{2!}{2}}+\frac{\frac{3}{8}x^3}{\underset{3!}{6}}-\frac{\frac{15}{16}x^4}{\underset{4!}{24}}$$

$$1+\frac{1}{2}(\cancel{x}^2)-\frac{\frac{1}{4}}{2}(\cancel{x}^2)^2+\frac{\frac{3}{8}(\cancel{x}^2)^3}{6}-\frac{\frac{15}{16}(x^2)^4}{24}$$

$$\boxed{P_4(x)=1+\underset{0}{}\frac{1}{2}x^2\underset{1}{}-\frac{1}{8}x^4\underset{2}{}}+\underset{3}{\frac{1}{16}x^6}-\underset{4}{\frac{5}{128}x^8}$$

$$1+\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\frac{2k+3}{2^k}}{k!}x^{2k} \qquad (2x)^{2k} \qquad 2^{2k} \qquad x^{2k}$$

(25) $e^{x/2}$; $x=0$ $\quad e^{0/2}=1$

$\frac{1}{2}e^{x/2}$ $\quad \frac{1}{2}e^{0/2}=\frac{1}{2}$

$\frac{1}{4}e^{x/2}$ $\quad \frac{1}{4}e^{0/2}=\frac{1}{4}$

$$P_2(x)=\underset{0}{1}+\underset{1}{\frac{1}{2}x}+\frac{\frac{1}{4}x^2}{\underset{2}{2!}}+\dots+\frac{\frac{1}{2^k}}{k!}x^k+\dots$$

$$P_2(x)=1+\frac{1}{2}x+\frac{1}{8}x^2+\dots+\frac{1}{2^k k!}x^k+\dots\frac{1}{2^k\cdot k!}x^k$$

(b) $\frac{e^{x}-1}{x}$

$$\frac{x^6}{x^2} = x^4$$

9.2

(b) $e^x \quad e^0 = 1$ $\qquad \frac{1}{0!} + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{1}{k!}x^k + \dots$

$e^x \quad e^0 = 1$

$e^x \quad e^0 = 1$

$$\frac{e^x - 1}{x} \qquad 1 + \frac{x}{2} + \frac{x^2}{6} + \dots + \frac{1}{k!}x^{k-1} + \dots$$

$$\frac{1}{(k+1)!}x^k$$

(c) $\cancel{g'(1) = e^1 - 1}$

$$g(x) = \frac{e^x - 1}{x} \quad \begin{matrix} f \\ g \end{matrix}$$

$$g'(x) = \frac{e^x \cdot x - (e^x - 1)}{x^2}$$

$$g'(1) = \frac{e^1 \cdot 1 - e^1 + 1}{1^2}$$

$= 1$

$$\frac{d}{dx} \frac{1}{(n+1)!} x^n$$

$$\frac{n}{(n+1)!} x^{n-1}$$

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1$$

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27 (c) $P_3(x) = 1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{1}{16}x^6$

$$\int P_3(x) = x + \frac{1}{6}x^3 - \frac{1}{40}x^5 + \frac{1}{112}x^7 + 5$$