9.4

(11) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{10^n}$ $\quad \lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{10^{n+1}} \cdot \frac{10^n}{(x-2)^n} \right| < 1$

$\lim_{n\to\infty} \left| \frac{(x-2)}{10} \right| < 1$

$10 \left| \frac{x-2}{10} \right| < 1 \cdot 10$

$|x-2| < 10$ $\quad \boxed{R = 10}$

$x - 2 < 10$ $\quad x - 2 > -10$

$x < \cancel{8}\,12$ $\quad x > -8$

$-8, 12$

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(23) $\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{4^n}$ $\quad \lim_{n\to\infty} \left| \frac{(x-1)^{2n+2}}{4^{n+1}} \cdot \frac{4^n}{(x-1)^{2n}} \right| < 1$

$\left(\frac{(x-1)^2}{4} \right)^n$

$\cancel{2n}$ $\quad \sum_{n=0}^{\infty} \frac{(-1-1)^{2n}}{4^n}$ $\quad \lim_{n\to\infty} \left| \frac{(x-1)^2}{4} \right| < 1$

$2(\cancel{x}+1)$ $\quad \sum_{n=0}^{\infty} \frac{(-2)^{2n}}{4^n}$ $\quad \left| \frac{(x-1)^2}{4} \right| < 1$ $\quad \sum_{n=0}^{\infty} \frac{(3-1)^{2n}}{4^n}$

$\sum_{n=0}^{\infty} \frac{(-1)^{2n}(2)^{2n}}{4^n}$ $\quad |(x-1)^2| < 4$ $\quad \sum_{n=0}^{\infty} \frac{\cancel{2^{2n}}}{\cancel{4^n}}$

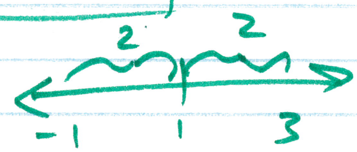
$\sum_{n=0}^{\infty} \frac{(-1)^{2n}\cancel{((2)^2)^n}}{\cancel{4^n}}$ $\quad |x-1| < 2$ $\quad \sum_{n=0}^{\infty} 1$

$|x-c| < R$

$\boxed{(-1, 3)}$

$\sum_{n=0}^{\infty} 1$ $\quad \lim_{n\to\infty} 1 = 1$

DIVERGE



9.4

23 $$\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{4^n} = \sum_{n=0}^{\infty} \frac{((x-1)^2)^n}{4^n} = \sum_{n=0}^{\infty} \left(\frac{(x-1)^2}{4}\right)^n$$

9.4

25 $\lim_{n\to\infty} \left| \dfrac{\left(\frac{\sqrt{x}}{2} - 1\right)^{n+1}}{\left(\frac{\sqrt{x}}{2} - 1\right)^{n}} \right| < 1$ $-5^2 < 1^2$

$\lim_{n\to\infty} \left| \frac{\sqrt{x}}{2} - 1 \right| < 1$ $25 < 1$

$\left| \frac{\sqrt{x}}{2} - 1 \right| < 1$

$\left| \frac{1}{2}(\sqrt{x} - 2) \right| < 1$

$\left| \sqrt{x} - 2 \right| < 2$

$\sqrt{x} - 2 < 2$ $\sqrt{x} - \cancel{2} > -2$

$+2 \quad +2$ $\cancel{+2} \quad +2$

$\sqrt{x}^2 < 4^2$ $\sqrt{x}^2 > 0^2$

$x < 16$ $x > 0$

$\boxed{(0, 16)}$

$\sum_{n=0}^{\infty} \left(\frac{\sqrt{0}}{2} - 1\right)^n$ $\sum_{n=0}^{\infty} \left(\frac{\sqrt{16}}{2} - 1\right)^n$

$\sum_{n=0}^{\infty} (-1)^n$ $\sum_{n=0}^{\infty} 1^n$

$\sum_{n=0}^{\infty} \left(\frac{\sqrt{x}}{2} - 1\right)^n = \dfrac{1}{1 - \left(\frac{\sqrt{x}}{2} - 1\right)}$ $\left| \frac{\sqrt{x}}{2} - 1 \right| < 1$

$\frac{\sqrt{x}}{2} < 2$

$\frac{\sqrt{x}}{2} - 1 > -1$ $\frac{\sqrt{x}}{2} > 0$ $x < 16$ $\sqrt{x} < 4$

9.4

(13) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n\sqrt{n}\,3^n}$

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)\sqrt{n+1}\,3^{n+1}} \cdot \frac{n\sqrt{n}\,3^n}{x^n} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{x\; n\sqrt{n}}{3\,(n+1)\sqrt{n+1}} \right| < 1 \qquad \frac{n^{3/2}}{1n^{3/2}\ldots}$$

$$\left| \frac{x}{3} \right| < 1$$

$$|x| < 3 \qquad \boxed{R = 3}$$

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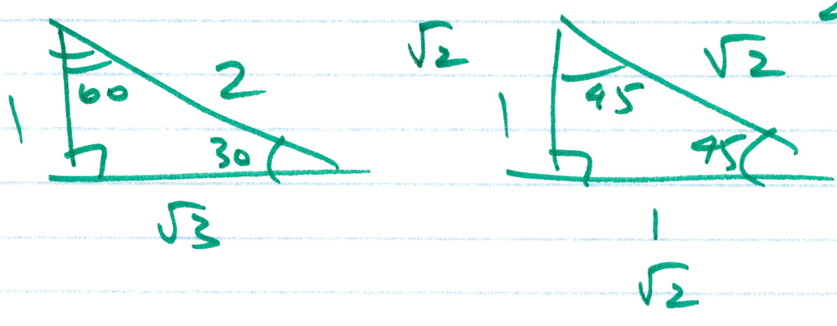
(33) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n+1}$ CONV $\qquad \displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n} = \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ CON.

$$\lim_{n\to\infty} \left| \frac{2^n}{3^n+1} \cdot \frac{3^n}{2^n} \right| = \lim_{n\to\infty} \left| \frac{\cancel{2^n}\cdot\cancel{3^n}}{\cancel{3^{n+1}}\cdot\cancel{2^n}} \right| = \cancel{\frac{1}{3}}$$

LIMIT COMPARISON $\displaystyle\lim_{n\to\infty} \left| \frac{3^n}{3^n+1} \right| = 1$

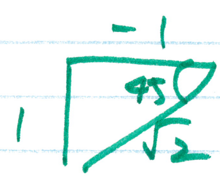
$\dfrac{2^n}{3^n+1} < \left(\dfrac{2}{3}\right)^n$ DIRECT COMPARISON

CONV.



2 $\qquad$ 225 $\quad 225°$

$\begin{array}{r} 225 \\ -180 \\ \hline 45 \end{array}$ $\qquad \dfrac{5\pi}{4} - \pi = \dfrac{\pi}{4}$



$180 = \pi$