9.5

(19) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{10^n}{n^{10}}$

~~$\frac{10^{n+1}}{(n+1)^{10}} < \frac{10^n}{n^{10}}$~~

$\lim_{n\to\infty} \frac{10^n}{n^{10}} = \infty$

$\boxed{\text{DIVERGES}}$

(23) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$

ABSOLUTELY : $\sum_{n=1}^{\infty} \frac{1+n}{n^2}$ $\quad \sum_{n=1}^{\infty} \frac{1}{n}$

DIVERGES

CONDITIONALLY : ALTERNATING SERIES TEST

1) $\lim_{n\to\infty} \frac{1+n}{n^2} = 0$

$\lim_{n\to\infty} \frac{\frac{1+n}{n^2}}{\frac{1}{n}}$ $\frac{1+n}{n^2} \cdot \frac{n}{1} = \frac{n+n^2}{n^2} = 1$

LIMIT COMPARISON TEST

2) $\frac{1+(n+1)}{(n+1)^2} < \frac{1+n}{n^2}$

$\lim_{n\to\infty} \frac{1+n}{\cancel{n^2}} \cdot \frac{\cancel{n^2}}{1} = \infty$

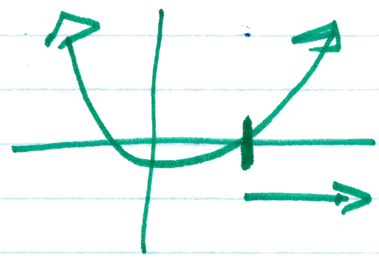
$n^2(n+2) < (1+n)(n+1)^2$

$\frac{1}{2} - \frac{3}{4}$

$n^3+2n^2 < (1+n)(n^2+2n+1)$

$\frac{4}{9}$

$n^3+2n^2 < n^2+2n+1+n^3+2n^2+n$



$\cancel{n^3}+2n^2 < \cancel{n^3}+3n^2+3n+1$

$0 < n^2+3n+1$

$\frac{-3 \pm \sqrt{9-4}}{2}$

$n > \frac{-3+\sqrt{5}}{2}$ ✓

MAXIMUM ERROR : $\frac{1+100}{100^2} = \frac{101}{100^2} = \frac{101}{10000} = .0101$

$\frac{1+n}{n^2}$ (100 → n)

$$\sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^n$$

9.5

24 $$\sum_{n=1}^{\infty} (-1)^{n+1} (.1)^n$$

ABSOLUTELY: $$\sum_{n=1}^{\infty} (.1)^n$$

CONVERGENT

ABSOLUTELY CONVERGENT

MAX ERROR $- .1^{100}$ $\left(\frac{1}{10}\right)^{100}$ $\frac{1^n}{10^n}$ $\frac{1}{10^n}$

$\frac{1}{n^p}$ $\frac{1}{n^{p+1}}$ $\frac{1}{n^{p-1}}$ $\frac{1}{n^{2p}}$

$p > 1$ $p > 0$ $p > 2$ $p > \frac{1}{2}$

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41 $$\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}\,3^n}$$

$$\lim_{n \to \infty} \left| \frac{x^{n+1}}{(n+1)\sqrt{n+1}\,3^{n+1}} \cdot \frac{3^n\, n\sqrt{n}}{x^n} \right| < 1$$

$$\lim_{n \to \infty} \left| \frac{x \quad n \sqrt{n}}{3 \quad (n+1)\sqrt{n+1}} \right| < 1$$

$$\left| \frac{x}{3} \right| < 1$$

$$|x| < 3$$

$$[-3, 3]$$

at $x = -3$ $$\sum_{n=1}^{\infty} \frac{(-3)^n}{n\sqrt{n}\,3^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (3)^n}{n\sqrt{n}\,3^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/2}}$$

CONVERGES (p-series)

at $x = 3$ $$\sum_{n=1}^{\infty} \frac{(3)^n}{n\sqrt{n}\,3^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$ CONVERGES