2.3

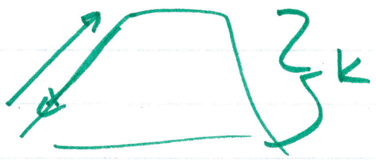
(29) $f(x) = \frac{x-4}{\sqrt{x}-2} = \frac{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)}{\cancel{\sqrt{x}-2}}$

$= \boxed{\sqrt{x}+2}$

$\frac{x^2-4}{x-2} = \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$

(45) $x = x^4 - 1$

(78)



$s(t)$ TRIP UP

$r(t)$ TRIP DOWN

$s(0) = 0$

$s(20) = k$

$f(t) = s(t) - r(t) = 0$

$f(0) = 0 - k = -k < 0$

$f(20) = k - 0 = k > 0$

BY IVT, SINCE POSITION VS. TIME IS A CONTINUOUS FUNCTION, THE FUNCTION TAKES ON ALL VALUE BETWEEN $f(0)$ & $f(20)$. SINCE $f(0) < 0$ AND $f(20) > 0$, THERE IS SOME c ON $[0, 20]$ SUCH THAT $f(c) = 0$. IF $f(c) = 0$, THEN AT c $s(c) = r(c)$ AND HE IS AT THE SAME PLACE AT THE SAME TIME. Q.E.D.

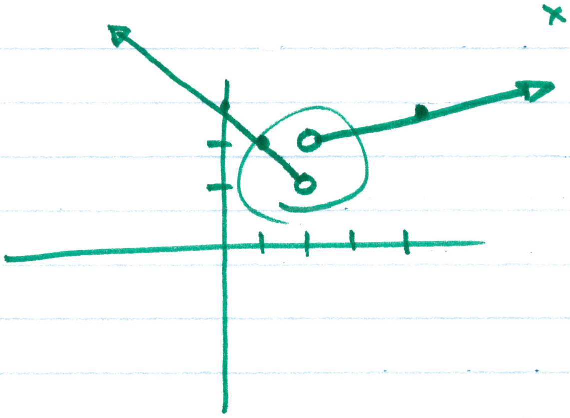
2.3

$\frac{1}{2}x+1$

19 $f(x) = \begin{cases} 3-x, & x<2 \\ \frac{x}{2}+1, & x>2 \end{cases}$ $\qquad x=2$

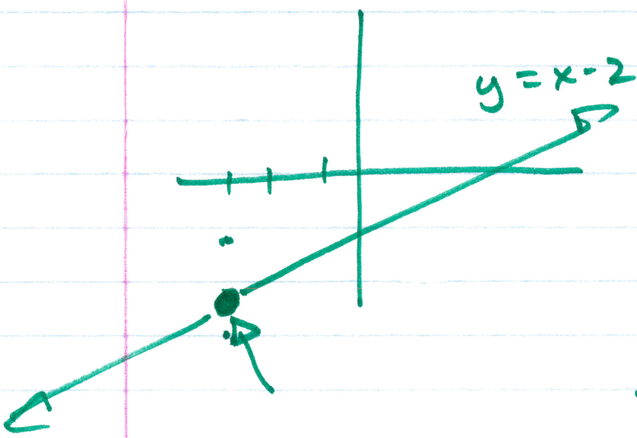
$\lim_{x\to 2^-} 3-x = 3-2 = 1$

$\lim_{x\to 2^+} \frac{x}{2}+1 = \frac{2}{2}+1 = 2$



non-removable

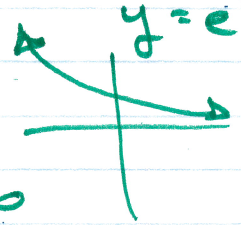
removable $\qquad \dfrac{(x-2)\cancel{(x+3)}}{\cancel{x+3}} \qquad x=-3$ removable



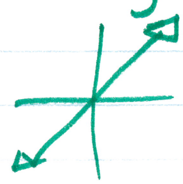
$f(x) = \begin{cases} \dfrac{(x-2)(x+3)}{x+3}, & x\neq -3 \\ -5 & x=-3 \end{cases}$

$x-2$

$-3-2=-5$



51 $e^{-x}=x$ has at least one solution



$e^{-x}-x=0$ $\qquad \lim_{x\to-\infty} e^{-x}-x > 0 \quad \infty$

$\lim_{x\to\infty} e^{-x}-x < 0 \quad -\infty$

CONSIDER $e^{-x}-x=0$. IF THIS EQUATION HAS SOLUTIONS, THEN $e^{-x}=x$. SINCE $\lim_{x\to-\infty} e^{-x}-x>0$, $e^{-x}-x$ HAS POSITIVE VALUES. SINCE $\lim_{x\to\infty} e^{-x}-x<0$, $e^{-x}-x$ HAS NEGATIVE VALUES. SINCE $e^{-x}-x$ IS A CONTINUOUS FUNCTION, WE HAVE $e^{-x}-x$ TAKING ON ALL VALUES, INCLUDING 0, BY THE IVT.