3.1

5 $f(x) = \frac{1}{x}$, $a = 2$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to a^2} \frac{\left(\frac{1}{x} - \frac{1}{2}\right)}{(x-2)} \cdot \frac{2x}{2x} = \lim_{x \to 2} \frac{2 - x}{2x(x-2)} = \lim_{x \to 2} \frac{-\cancel{(x-2)}}{2x\cancel{(x-2)}}$$

$$\lim_{x \to 2} \frac{-1}{2x} = \boxed{-\frac{1}{4}}$$

1 $f(x) = \frac{1}{x}$, $a = 2$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

3 $f(x) = 3 - x^2$, $a = -1$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\lim_{h \to 0} \frac{\left[3 - (x+h)^2\right] - \left[3 - x^2\right]}{h}$$

$$\lim_{h \to 0} \frac{\left[\cancel{3} - \cancel{x^2} - 2xh - h^2\right] - \left[\cancel{3} - \cancel{x^2}\right]}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(-2x - h)}{\cancel{h}}$$

$$\lim_{h \to 0} -2x - h = -2x$$

$$-2(-1) = 2$$

3.1

7 $f(x) = \sqrt{x+1}$, $a = 3$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x - 3} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2} = \lim_{x \to 3} \frac{x + 1 - 4}{(x-3)(\sqrt{x+1} + 2)}$$

$$\lim_{x \to 3} \frac{\cancel{x-3}}{(\cancel{x-3})(\sqrt{x+1} + 2)} = \lim_{x \to 3} \frac{1}{\sqrt{x+1} + 2} = \boxed{\frac{1}{4}}$$

19 $y = x^3$, $(1, 1)$

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$$

$$\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

1 $()^0$

1 1 $()^1$

1 2 1 $()^2$

1 3 3 1 $()^3$

1 4 6 4 1

$(x+h)(x+h)(x+h)$

$$\lim_{h \to 0} \frac{[\cancel{x^3} + 3x^2h + 3xh^2 + h^3] - \cancel{x^3}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$3x^2$

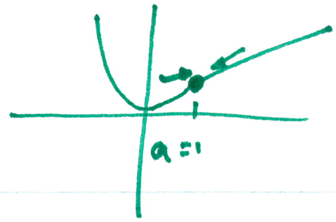
$3(1)^2 = 3$

$y = 3(x-1) + 1$

$y - 1 = 3(x-1)$

(b) $m = -\frac{1}{3}$

$$\boxed{y = -\frac{1}{3}(x-1) + 1}$$



3.1

$a=1$

42 $f(x)=\begin{cases} x^2, & x \le 1 \\ 2x, & x > 1 \end{cases}$

$$\lim_{h\to 0^-} \frac{(1+h)^2-1}{h}$$

$$\lim_{h\to 0^-} \frac{\cancel{1}+2h+h^2\cancel{-1}}{h}$$

$$\lim_{h\to 0^-} \frac{\cancel{h}(2+h)}{\cancel{h}}$$

$$\lim_{h\to 0^-} 2+h = 2$$

$$\lim_{h\to 0^+} \frac{2(1+h)-1}{h}$$

$$\lim_{h\to 0^+} \frac{2+2h-1}{h}$$

$$\lim_{h\to 0^+} \frac{1+2h}{h}$$

$$\lim_{h\to 0^+} \frac{1}{h} + \frac{2\cancel{h}}{\cancel{h}}$$

$$\infty + 2 = \infty$$

$\frac{1}{0}$ $\quad 0\overline{)1}$

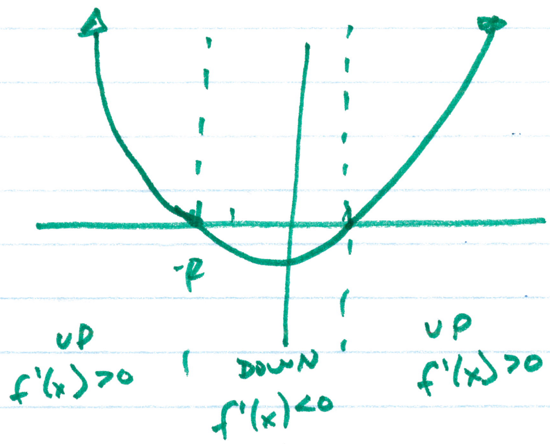
3.1

(17) $f(2)=3, \quad f'(2)=5$

(a) TANGENT $\quad y-3=5(x-2)$

(b) NORMAL $\quad y-3=-\frac{1}{5}(x-2)$

(22)



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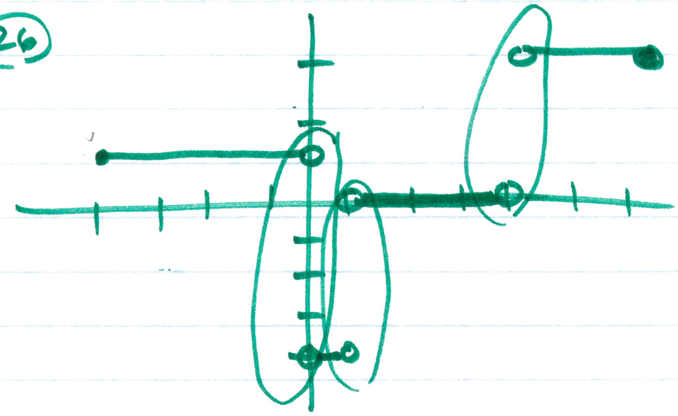


(42) $f(x)=\begin{cases} x^2, & x \le 1 \\ 2x, & x>1 \end{cases}$

$$\lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$\lim_{h\to 0} 2x+h = 2x$$

$f'(x)=\begin{cases} \text{(a) } f'(x) \text{ for } x<1 & 2x \\ \text{(b) } f'(x) \text{ for } x>1 & 2 \end{cases}$

$$f'(x)=\begin{cases} 2x, & x<1 \\ 2, & x>1 \end{cases}$$

(c) $\lim_{x\to 1^-} f'(x) = 2$

(d) $\lim_{x\to 1^+} f'(x) = 2$

(e) $\lim_{x\to 1} f'(x) = 2$

(f) $\lim_{h\to 0^-} \frac{(1+h)^2-1^2}{h}$

$$\lim_{h\to 0^-} \frac{\cancel{1}+2h+h^2-\cancel{1}}{h}$$

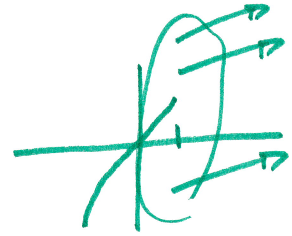
$$\lim_{h\to 0^-} \frac{\cancel{h}(2+h)}{\cancel{h}} = 2$$

(g) $\lim_{h\to 0^+} \frac{2(1+h)-1^2}{h}$

$$\lim_{h\to 0^+} \frac{2+2h-1}{h}$$

$$\lim_{h\to 0^+} \frac{1+2h}{h}$$

$$\lim_{h\to 0^+} \frac{1}{h}+\frac{2\cancel{h}}{\cancel{h}} = \infty$$



3.1

(44) $f(x) = \begin{cases} x^3, & x \le 1 \\ 3x+k, & x > 1 \end{cases}$

$(1+2h+h^2)(1+h)$

$1+2h+h^2+h+2h^2+h^3$

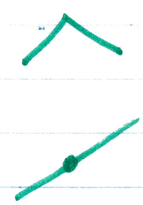
CONTINUOUS @ $x=1$

$x^3 = 3x+k$

$(1)^3 = 3(1)+k$

$1 = 3+k$

$\boxed{-2 = k}$



DIFFERENTIABLE @ $x=1$

$\lim_{h\to 0^-} \frac{(1+h)^3 - 1^3}{h}$

$\lim_{h\to 0^-} \frac{\cancel{1}+3h+3h^2+h^3 \cancel{-1}}{h}$

$\lim_{h\to 0^-} \frac{\cancel{h}(3+3h+h^2)}{\cancel{h}}$

$\lim_{h\to 0^-} 3+3h+h^2 = 3$

$\lim_{h\to 0^+} \frac{3(1+h)+k-1}{h}$

$\lim_{h\to 0^+} \frac{3+3h+k-1}{h}$

$\lim_{h\to 0^+} \frac{2+3h+k}{h}$ $\boxed{k=-2}$

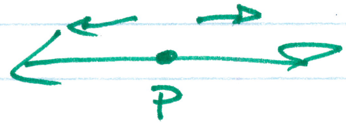
3.1

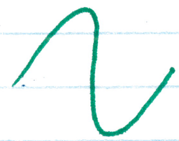
(7) $$\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2}$$

$$\lim_{x \to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$\lim_{x \to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$$

$$\lim_{x \to 3} \frac{1}{\sqrt{x+1}+2} = \boxed{\frac{1}{4}}$$





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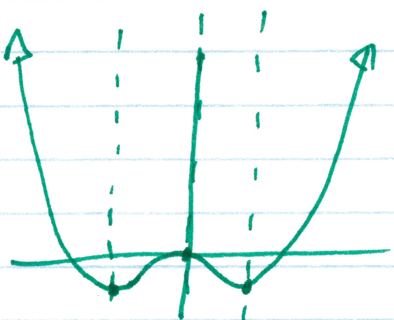

DECR | INCR | DECR | INCR

f' − | f' | f' | f'

+ − +