3.6

(35) $f(u) = \cot \frac{\pi u}{10}$ $\qquad u = g(x) = 5\sqrt{x} = 5x^{1/2}$, $x = 1$

$v = \frac{\pi u}{10} = \frac{\pi}{10} u$ $\qquad du = \frac{5}{2} x^{-1/2} = \frac{5}{2\sqrt{x}}$

$dv = \frac{\pi}{10} du$

$f(v) = \cot v$

$f'(v) = -\csc^2 v \, dv$

$f'(u) = -\csc^2 \left(\frac{\pi}{10} u\right) \left[\frac{\pi}{10}\right] du$

$f'(x) = -\csc^2 \left(\frac{\pi}{10} \cdot 5\sqrt{x}\right) \left[\frac{\pi}{10}\right] \left[\frac{5}{2\sqrt{x}}\right]$

$f'(1) = -\frac{\pi}{4}$

$f(x) = \cot \frac{\pi}{10}(5\sqrt{x})$ $\qquad u = \frac{\pi}{2} x^{1/2}$

$f(x) = \cot \frac{\pi}{2}\sqrt{x}$ $\qquad du = \frac{\pi}{4} x^{-1/2}$

$f(u) = \cot u$

$u = x^{1/2}$

$du = \frac{1}{2} x^{-1/2}$

$f(u) = \cot \frac{\pi}{2} u$

(11) $s = \frac{4}{3\pi} \sin 3t + \frac{4}{5\pi} \cos 5t$

$u = 3t \rightarrow du = 3$ $\qquad u = 5t \rightarrow du = 5$

$= \frac{4}{3\pi} \sin u \qquad \frac{4}{5\pi} \cos u$

$s' = \frac{4}{3\pi} \cos u \, du - \frac{4}{5\pi} \sin u \, du$

$= \frac{4}{3\pi} \cos 3t \, [3] - \frac{4}{5\pi} \sin 3t \, [5]$

$= \frac{4}{\pi} \cos 3t - \frac{4}{\pi} \sin 3t$

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⑤ $y = \left(\frac{\sin x}{1+\cos x}\right)^2$ ⟶ $y = u^2$

$u = \frac{\sin x}{1+\cos x}$ f, g

$y' = 2u\,du$

$du = \frac{\cos x(1+\cos x) - (-\sin x)(\sin x)}{(1+\cos x)^2}$

$y' = 2\left(\frac{\sin x}{1+\cos x}\right)\left[\frac{1}{1+\cos x}\right]$

$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ 1

$y' = \frac{2\sin x}{(1+\cos x)^2}$

$= \frac{\cos x + 1}{(1+\cos x)^2}$

$= \frac{1}{1+\cos x}$

$\sqrt{x^2} = \sqrt{4}$

no solutions

㉗ $r = \sqrt{\theta \sin\theta} = (\theta \sin\theta)^{1/2}$

$u = \theta \sin\theta$ f, g

$du = 1\sin\theta + \theta\cos\theta$

$r = u^{1/2}$

$r' = \frac{1}{2}u^{-1/2}\,du$

$= \frac{1}{2}(\theta\sin\theta)^{-1/2}[\sin\theta + \theta\cos\theta]$

$= \frac{\sin\theta + \theta\cos\theta}{2\sqrt{\theta\sin\theta}}$

3.6

(21) $y = \sin^2(3x-2)$

$y = (\sin(3x-2))^2$

$u = \sin(3x-2)$

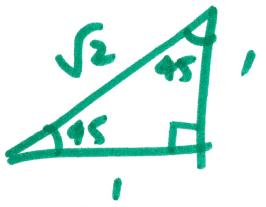
$du = \cos(3x-2)\ [3]$

$y = u^2$

$y' = 2u\,du$

$y' = 2\sin(3x-2)\left[3\cos(3x-2)\right]$

$y' = 6\sin(3x-2)\cos(3x-2)$



3.6

27 $r = \sqrt{\theta \sin\theta} = (\theta \sin\theta)^{1/2}$

$f = \theta \sin\theta$, $g = \theta$

$\frac{1}{2}(\theta \sin\theta)^{-1/2}[1 \sin\theta + \theta \cos\theta]$

$f' = 1 \sin\theta$, $g' = \theta \cos\theta$

$$\frac{\sin\theta + \theta\cos\theta}{2\sqrt{\theta \sin\theta}}$$

55 $y = 2\tan(\pi x/4)$ @ $x = 1$ $(1, 2)$

$y = 2\tan(\pi \cdot 1/4)$

$y = 2 \cdot 1 = 2$

$\frac{\pi}{4} x$

$y' = 2\sec^2(\pi x/4)\left[\frac{\pi}{4}\right]$

$y'(1) = 2\sec^2(\pi \cdot 1/4)\left[\frac{\pi}{4}\right]$

$= 2(\sqrt{2})^2\left[\frac{\pi}{4}\right]$

$= 2 \cdot 2\left[\frac{\pi}{4}\right] = \pi$

TANGENT: $y - 2 = \pi(x - 1)$

NORMAL: $y - 2 = -\frac{1}{\pi}(x - 1)$

$\frac{\pi}{10}u$

3.6

35 $f(u) = \cot \frac{\pi u}{10}$ $\quad u = 5\sqrt{x}$ $\quad x > 1$

$u = 5x^{1/2}$

$du = \frac{5}{2}x^{-1/2}$

$f'(u) = -\csc^2\left(\frac{\pi u}{10}\right)\left[\frac{\pi}{10}\,du\right]$

$f'(x) = -\csc^2\left(\frac{\pi}{10}\cdot 5\sqrt{x}\right)\left[\frac{\pi}{10}\cdot\frac{5}{2}x^{-1/2}\right]$

$f'(1) = -\csc^2\left(\frac{\pi}{10}\cdot 5\sqrt{1}\right)\left[\frac{\pi}{10}\cdot\frac{5}{2}(1)^{-1/2}\right]$

$= -\csc^2\left(\frac{\pi}{2}\right)\left[\frac{\pi}{4}\right]$

$\boxed{= -\frac{\pi}{4}}$

$f(x) = \cot \frac{\pi}{10}\cdot 5\sqrt{x}$

$= \cot \frac{\pi}{2}\sqrt{x}$ $\xrightarrow{?}$ $u = 5\sqrt{x}$

54 $y = \sin mx \qquad (0,0)$

$y' = \cos(mx)\,[m]$

$y' = \cos(m\cdot 0)\,[m] = m$

$\boxed{y = mx}$

$\frac{3}{(2x+1)^{1/2}}$

3.6

(19) $y = \frac{3}{\sqrt{2x+1}} = 3(2x+1)^{-\frac{1}{2}}$

$y' = -\frac{3}{2}(2x+1)^{-\frac{3}{2}}[2]$

$= \boxed{-3(2x+1)^{-\frac{3}{2}} = \frac{-3}{\sqrt{(2x+1)^3}}}$

$f(x) = \sin x$ $\quad \sin x^3$

$g(x) = x^3$ $\quad \cos x^3 \; [3x^2]$

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(37) $f(u) = \frac{2u}{u^2+1}$ (f, g) $\quad u = 10x^2 + x + 1$ $\quad (x=0)$

$du = 20x + 1$

$f'(u) = \frac{2\,du(u^2+1) - 2u\,du(2u)}{(u^2+1)^2}$ $\quad (g', f')$

$u(0) = 1$

$du(0) = 1$

$f'(0) = \frac{2(1)(2) - 2(1)(1)(2(1))}{(2)^2}$

$= \frac{4-4}{4} = 0$

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(62) ~~illegible~~

(63) $s = \sqrt{1+4t} = (1+4t)^{1/2}$

$v = s' = \frac{1}{2}(1+4t)^{-1/2}[4]$

$= 2(1+4t)^{-1/2} = \frac{2}{\sqrt{1+4t}}$

$s'(6) = 2(1+4(6))^{-1/2} = \boxed{\frac{2}{5}\ m/s}$

$a = v' = s'' = -(1+4t)^{-3/2}[4]$

$= -4(1+4t)^{-3/2} = \frac{-4}{\sqrt{(1+4t)^3}}$

$s''(6) = -4(1+4(6))^{-3/2} = \boxed{\frac{-4}{125}\ m/s^2}$