6.4

(30) $\ln|y-y_c| = kt + C$

$\ln|35-65| = k(10) + C$

$\ln 30 = 10k + C$

$-10k \quad -10k$

$\boxed{\ln 30 - 10k = C}$

$\ln 30 - 10\left(\frac{\ln\frac{1}{2}}{10}\right) = C$

$\ln 30 - \ln\frac{1}{2} = C$

$\ln 60 = C$

$\ln|50-65| = k(20) + C$

$\ln 15 = 20k + C$

$\ln 15 = 20k + \ln 30 - 10k$

$-\ln 30 \qquad -\ln 30$

$\frac{\ln\frac{1}{2}}{10} = \frac{10k}{10}$

$\ln|y-y_c| = \frac{\ln\frac{1}{2}}{10}t + \ln 60$

$\ln|y-65| = \frac{\ln\frac{1}{2}}{10}\cdot 0 + \ln 60$

$\ln|y-65| = \ln 60$

$y - 65 = 60$

$y = 125$

$y - 65 = -60$

$\boxed{y = 5^\circ F}$

$|x+3| = 4$

$\boxed{x+3=4}$ $\qquad$ $\boxed{x+3=-4}$

$x = 1$ $\qquad$ $x = -7$

6.4

(22) $\frac{y}{C} = \frac{Ce^{kt}}{C} \longrightarrow y = Ce^{\frac{\ln\frac{1}{2}}{65}t}$

$\frac{1}{2} = e^{kt}$

$\frac{1}{2} = e^{k(65)}$

$\boxed{\frac{\ln\frac{1}{2}}{65} = \frac{65k}{65}}$

(19) 4.75% (b) monthly (d) continuously

(b) $\frac{A}{P} = \frac{P(1+\frac{r}{n})^{nt}}{P}$

$2 = (1+\frac{r}{n})^{nt}$

$2 = (1+\frac{.0475}{12})^{12t}$

$\ln 2 = \ln(1+\frac{.0475}{12})^{12t}$

$\frac{\ln 2}{12\ln(1+\frac{.0475}{12})} = \frac{12t\ln(1+\frac{.0475}{12})}{12\ln(1+\frac{.0475}{12})}$

(d) $\frac{A}{P} = \frac{Pe^{rt}}{P}$

$2 = e^{rt}$

$2 = e^{.0475t}$

$\frac{\ln 2}{.0475} = \frac{.0475t}{.0475}$

$\frac{\ln 2}{.0475} = t$

6.4

(31) $\ln|y-y_c| = kt+C$

$\ln|90-20| = k(0)+C$

$\ln 70 = C$ ✓

$\ln|60-20| = k(10)+C$

$\ln 40 = 10k + \ln 70$

$-\ln 70 \qquad -\ln 70$

$\frac{\ln \frac{40}{70}}{10} = \frac{10k}{10}$

$\frac{\ln \frac{4}{7}}{10} = k$ ✓

(a) $\ln|35-20| = \frac{\ln \frac{4}{7}}{10} t + \ln 70$

$\ln 15 = \frac{\ln \frac{4}{7}}{10} t + \ln 70$

$-\ln 70 \qquad -\ln 70$

$\frac{10}{\ln \frac{4}{7}} \ln \frac{15}{70} = \frac{\ln \frac{4}{7}}{10} t \cdot \frac{10}{\ln \frac{4}{7}}$

$$\frac{10 \ln \frac{3}{14}}{\ln \frac{4}{7}} = t$$

27.53 min

-10

17.53 min.

(b) $\ln|y-y_c| = kt+C$

$\ln|90--15| = k(0)+C$

$\ln 105 = C$ ✓

$\ln|35--15| = \frac{\ln \frac{4}{7}}{10} t + \ln 105$

$\ln 50 = \frac{\ln \frac{4}{7}}{10} t + \ln 105$

$-\ln 105 \qquad -\ln 105$

$\frac{10}{\ln \frac{4}{7}} \ln \frac{50}{105} = \frac{\ln \frac{4}{7}}{10} t \cdot \frac{10}{\ln \frac{4}{7}}$

$$\frac{10 \ln \frac{10}{21}}{\ln \frac{4}{7}} = t$$

6.4

⑦ $\frac{dy}{dx} = \cos x \, e^{y+\sin x}$

$x^2 \cdot x^5 = x^{2+5}$

$\frac{dy}{dx} = \frac{\cos x \, e^y e^{\sin x}}{e^y}$

$-\int e^{-y} \, dy = \int \cos x \, e^{\sin x} \, dx$

du $\quad$ du

$u = -y$	$u = \sin x$
$du = -dy$	$du = \cos x \, dx$
$-\int e^u \, du$	$\int e^u \, du$
$-e^u$	$e^u + C$

$-e^{-y} = e^{\sin x} + C$

$-e^{-0} = e^{\sin 0} + C$

$-1 = 1 + C$

$-2 = C$

$-e^{-y} = e^{\sin x} - 2$

$\ln e^{-y} = \ln(-e^{\sin x} + 2)$

$-y = \ln(-e^{\sin x} + 2)$

$\boxed{y = -\ln(-e^{\sin x} + 2)}$

6.4

(21) $\frac{dy}{dt} = -.0077y$ (k)

$\frac{dy}{dt} = ky \rightarrow y = Ce^{kt}$

$\frac{y}{C} = \frac{Ce^{-.0077t}}{C}$

$\frac{1}{2} = e^{-.0077t}$

$\ln \frac{1}{2} = \ln e^{-.0077t}$

$\frac{\ln \frac{1}{2}}{-.0077} = \frac{-.0077t}{-.0077}$

6.4

⑦ $\frac{dy}{dx} = (\cos x)e^{y+\sin x}$

$\frac{dy}{dx} = \frac{(\cos x)e^{y}e^{\sin x}}{e^{y}}$

$\int e^{-y}\,dy = \int (\cos x)e^{\sin x}\,dx$ $\quad u = \sin x$, $du = \cos x\,dx$

$-e^{-y} = \int e^{u}\,du$

$-e^{-y} = e^{u} + C$

$-e^{-y} = e^{\sin x} + C$

$-e^{-0} = e^{\sin 0} + C$

$-1 = 1 + C$

$-2 = C$

$-e^{-y} = e^{\sin x} - 2$

$\ln(e^{-y}) = \ln(-e^{\sin x} + 2)$

$-y = \ln(-e^{\sin x} + 2)$

$y = (-)\ln(-e^{\sin x} + 2)$

$y = \ln(-e^{\sin y} + 2)^{-1}$

$y = \ln \frac{1}{-e^{\sin x} + 2}$

⑲ $A = P\left(1 + \frac{r}{n}\right)^{nt}$

- A: Amount at +
- P: Principal
- r: annual interest rate
- n: number of compoundings in a year
- t: time in years

6.4

(29) $y = y_0 e^{-kt}$ $\quad t = \frac{3}{k}$

$y = y_0 e^{-k\left(\frac{3}{k}\right)}$

$y = y_0 e^{-3}$

$e^{-3} \approx .0497$

(41) $p = Ce^{kh} \rightarrow$ & $p = Ce^{k(0)} = p_0$

$p = p_0 e^{kh}$

$p = 1013 e^{kh}$

$20 = 1013 e^{k(90)}$

$\frac{20}{1013} = e^{90k}$

$\frac{\ln \frac{20}{1013}}{90} = \frac{90k}{90}$

(a) $p = 1013 e^{\frac{\ln \frac{20}{1013}}{90} \cdot h}$

(b) $p = 1013 e^{\frac{\ln \frac{20}{1013}}{90} \cdot 50} = 1013 e^{\frac{\ln \frac{20}{1013}}{9} \cdot 5}$

(c) $900 = 1013 e^{\frac{\ln \frac{20}{1013}}{90} \cdot h}$